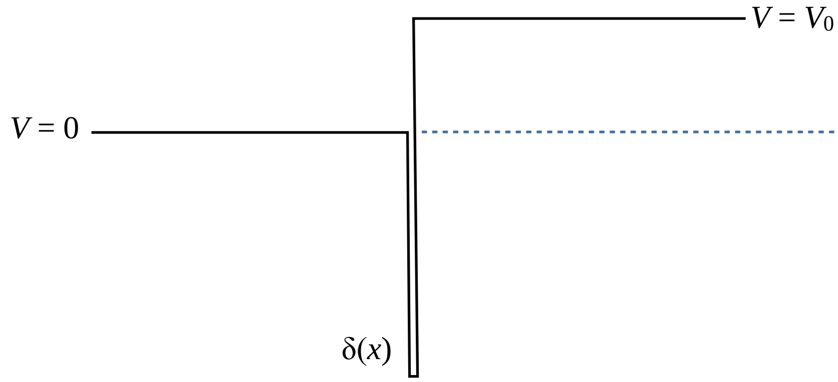
PHYS 5382 Introduction to Quantum Mechanics **Practice Test 2**

Time allowed 1 hour 15 min.

Resources: Griffiths textbook, calculator, formulas from the covers of Griffiths hardback

1. Consider a one-dimensional step-function potential with an attractive δ-function potential at its edge

$$V(x) = V_0 \ \Theta(x) - (\hbar^2 g / 2m)\ \delta(x)$$



(a) A wavefunction $A \exp(i\, k\, x)$ with $A = 1$ represents particles of energy $E > V_0$ incident from the left. Find the complete wavefunction (including transmitted and reflected parts) in each region in terms of k and q, where

$$k = \frac{\sqrt{2\, m\, E}}{\hbar} \qquad q = \frac{\sqrt{2\, m\, (E - V_0)}}{\hbar}$$

[10 points]

(b) Now consider energies $E < 0$. Find an implicit equation for allowed bound state energies E (you do not have to solve it) in terms of s and t, where

$$s = \frac{\sqrt{-2\, m\, E}}{\hbar} \qquad t = \frac{\sqrt{2\, m\, (V_0 - E)}}{\hbar}$$

[7 points]

2. A simple harmonic oscillator is in the state

$$\psi = N\,(\psi_0 + \lambda\, \psi_1)$$

where λ is a real parameter, and ψ_0 and ψ_1 are the first two orthonormal stationary states.

(a) Determine the normalization constant N in terms of λ. **[3 points]**

(b) Using raising and lowering operators (see Griffiths 2.69), calculate the uncertainty Δx in terms of λ. **[10 points]**