PHYS 5382 Introduction to Quantum Mechanics **Practice Test 3**

Time allowed 1 hour 15 min.

Resources: Griffiths textbook, calculator, formulas from the covers of Griffiths hardback

1. Consider a particle constrained to move in a circle, whose position is described by the azimuthal angle φ in the interval $[0, 2\pi]$. Assume that wavefunctions are periodic $\psi(\varphi) = \psi(\varphi + 2\pi)$, continuous, and have continuous first derivative $d\psi/d\varphi$.

   (a) Determine whether the following operator Q is Hermitian on this interval

   $$Q = \frac{d^2}{d\varphi^2}$$

   [3 points]

   (b) Find the normalized eigenfunctions and eigenvalues of Q and describe their degeneracy.
   [7 points]

   (c) Show that eigenfunctions with distinct eigenvalues are orthogonal to each other.
   [5 points]

2. A normalized wavefunction at time $t = 0$ is non-zero in a finite interval of the x-axis

$$\Psi(x,0) = \begin{cases} (2\lambda)^{-1/2} \exp(2\pi i\, x/\lambda) & -\lambda < x < \lambda \\ 0 & x < -\lambda,\ x > \lambda \end{cases}$$

where λ is a real parameter.

(a) Determine the momentum space wavefunction $\Phi(p,0)$ at $t = 0$ in terms of $\sin(p\lambda/\hbar)$
[7 points]

(b) Sketch $|\Phi|^2$ and describe what happens to the width and height of features as $\lambda \to \infty$.
[5 points]

(c) Which of the potentials studied in Griffiths Chapter 2 does this wavefunction belong to in the limit $\lambda \to \infty$ and what happens to Δp and Δx?
[3 points]