PHYS 5382 Introduction to Quantum Mechanics - Homeworks

Homework 1

1. Compute the de Broglie wavelength for an electron of energy 1 keV. If this electron were interacting with a hydrogen atom (look up the size of an atom), would classical mechanics suffice?

2. In kinetic theory of gases, atoms are modeled as point masses m with mean speed v related to temperature T by $m v^2 = 3 \mathrm{k} T$, where k is Boltzmann's constant. Assuming gas atoms travel several thousand Angstroms between collisions with each other, how cool would hydrogen gas need to be before quantum mechanics would have to be taken into consideration?

3. Given the initial wavefunction $\Psi(x, 0) = A\, x \exp(-k x)$ with $x > 0$ and $k > 0$, and $\Psi(x, 0) = 0$ for $x < 0$, what value must A take in terms of k in order that Ψ is normalized? Find $\langle x \rangle$, $\langle x^2 \rangle$, and σ.

Homework 2

1. A brief radio-wave pulse of photons is of duration 0.001 s. An individual photon might be anywhere in the pulse. Use the position-momentum uncertainty principle to estimate the uncertainty in the frequency of the light in the pulse.

2. Let $\Psi(x, t) = (A / (a^2 + x^2)) \exp(-i\, 2 \pi E t / h)$ be a normalized solution to Schrodinger's equation for constants A, a, and E.
(a) What is A in terms of a?
(b) What is the potential function $V(x)$?
(c) Evaluate $\Delta x\, \Delta p$. Is the uncertainty principle satisfied?

Homework 3

Let $\Psi(x, t = 0) = \psi_1/\sqrt{3} + \psi_2/\sqrt{4} + \psi_3\sqrt{(5/12)}$, where ψ_i is the i^{th} normalized stationary state solution of the infinite square well.
(a) Verify that Ψ itself is normalized
(b) Find $\Psi(x, t)$ and using the explicit wavefunctions express the probability density at time t as a real function.
(c) If E_n is the energy of the n^{th} normalized stationary state, what are the probabilities of measuring energy and getting the result E_1, or E_2, or E_3?
(d) What is $\langle H \rangle$ when written in terms of E_1 ?
(e) Compute $\langle x \rangle$ and $\langle p \rangle$

Homework 4

1. Show that explicit application of the lowering operator in terms of x and p operators on the n = 3 harmonic oscillator stationary state leads to the n=2 state.

2. Compute for the n=1 wavefunction of the harmonic oscillator the probability of finding the particle outside the classically allowed region. (To complete the calculation you will need to look up online the value of the "Error Function" integral.)

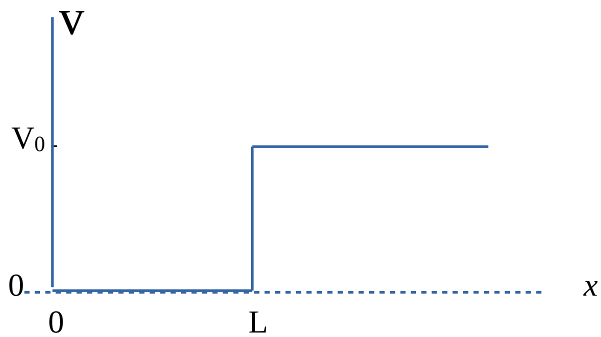
Homework 5

1. Derive the normalized stationary states and corresponding energies for a free particle of mass *m* that is constrained to move in a circle of circumference *L* (treat it like a free particle on the real line but with periodic boundary conditions $\psi(x) = \psi(x + L)$).

2. Show that the stationary states of the finite square well agree with those of a delta function potential in a suitable limit in which the well becomes infinitely deep and narrow.

3. Find the normalized stationary states and allowed bound state energies of the Schrodinger equation for a particle of mass *m* and energy $E < V_0$ in the semi-infinite potential well

$x < 0$, $V = \infty$



Homework 6

1. Show that for any function $f(x)$ and the momentum operator p_x, the operator

$$[f(x), p_x] = i\,\text{hbar}\,(df / dx)$$

[Hint: Have the operators "operate" on a wavefunction]

2. Consider the operator $d^2/d\varphi^2$ where φ is the azimuthal angle in polar coordinates and functions are subject to $f(\varphi) = f(\varphi + 2\pi)$. Is the operator Hermitian on such functions? Find its eigenfunctions and eigenvalues. What is the spectrum and is it degenerate?

Homework 7

1. Test the equation for time evolution of expectation values for observable Q = x and the wavefunction that is initially

$$\Psi\,(x, t{=}0) = A\,(\psi_1 + \psi_2),$$

where ψ_1 and ψ_2 are the first two stationary states of the infinite square well, by calculating $< [H\,, x] >$ and $d < x > / dt$ exactly.

[All the integrals can done using results in the back cover of Griffiths]

2. A 3-dimensional vector space is spanned by the orthonormal basis | 1 >, | 2 >, | 3 >.

Define

$$| a > = i\,| 1 > - 2\,| 2 >$$

$$| b > = 1 > + 2\,| 3 >$$

Find < a | b > and confirm that < b | a > = < a | b >*

Find all the matrix elements of the operator | a > < b |. Is it hermitian?

Homework 8

Determine $<r>$ and $<1/r>$ for the hydrogen atom in the (2,1,0) state.

Verify that the most probable $r = 4a$.

What is the probability of finding the electron between 3.9 and 4.1 Bohr radii from the nucleus?

Homework 9

1. The electron in a hydrogen atom occupies the combined spin and position state

$$R_{21}\,\left(\sqrt{(2/3)}\; Y^0_1\, \chi_+ \;+\; \sqrt{(1/3)}\; Y^1_1\, \chi_- \right)$$

What are the possible measurements and their probabilities for L^2, L_z, S^2, and S_z?

What is the probability density for finding the particle at radius r with spin up?

2. An electron is in the spin state χ where

$$\chi = A \begin{bmatrix} 1 + 2i \\ 2 \end{bmatrix}$$

What are the possible measurements and their probabilities for S_z, S_x, and S_y?

Homework 10

1. A beam of neutrons with spins along the positive *z*-axis suddenly enters a region where there is a uniform magnetic field B = 1.5 T (Tesla) in the positive *x*-direction. Ignoring the spatial degrees of freedom
   (a) find the spin state of the system at any time *t* after entering the B field region and interpret what is happening. *Hint: Use the Euler relation and corresponding Taylor expansions*

$$\exp(iA) = \cos A + i \sin A$$

$$\sum_{n=0}^{\infty} (iA)^n/n! = \sum_{n=\text{even}}^{\infty} (iA)^n/n! + \sum_{n=\text{odd}}^{\infty} (iA)^n/n!$$

   (b) What is the earliest time *t* that the system has evolved to have the spin completely flipped, so it is definitely along the negative *z*-direction? (Look up the gyromagnetic ratio for the neutron).

2. If total spin of a two-particle system is $\underline{\mathbf{S}} = \underline{\mathbf{S}}^{(1)} + \underline{\mathbf{S}}^{(2)}$, use the basic commutation relation for angular momentum operators $[S_x, S_y] = i\,\text{hbar}\,S_z$ and the fact that operators for different particles commute to show that $[S^2, \underline{\mathbf{S}}^{(1)}] = 2i\,\text{hbar}\,(\underline{\mathbf{S}}^{(1)} \text{ X } \underline{\mathbf{S}}^{(2)})$, where X means the vector cross product. *Hint: To shorten the calculation, do just the x component of the equation and argue from symmetry considerations for the y and z components.*