

IDEAS OF
MODERN PHYSICS

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Contents

- 1. Scientific Discovery**
 - 1.1 Numbers in Science
- 2. Classical Physics**
 - 2.1. Space, Time, Motion
 - 2.2. Gravity
 - 2.3. Electricity & Magnetism
 - 2.4. Light: Into the Modern Era
- 3. Special Relativity**
 - 3.1. Space, Time, Motion, Revisited
 - 3.2. Paradoxes (not)
 - 3.3. Energy & $E = mc^2$
 - 3.4. Space-Time
- 4. General Relativity**
 - 4.1. Equivalence Principle
 - 4.2. Time Dilation and Light Bending
 - 4.3. Curved Space-Time
 - 4.4. Structure of the Universe
- 5. Quantum Mechanics**
 - 5.1. Wave-Particle Duality of Light
 - 5.2. Probability & Uncertainty
 - 5.3. Matter Waves
 - 5.4. Quantum Measurements
- 6. Atoms**
 - 6.1. Structure and Properties

6.2. Quantized Energy

6.3. The Nucleus

6.4. Condensed Matter

7. Synthesis

7.1. Space-Time Revisited

7.2. Particles and Force-Fields

7.3. The Standard Model

7.4. Unsolved Mysteries

Preface

Through our present understanding of the physical universe at the most fundamental level, the aim of this book is to address questions no less than the following. Why is everything here? What is everything made of and how does it ultimately function? What will happen to everything in the future? What is real and what is imaginary? These are questions that transcend mere utility. If they do not interest you as questions *per se*, you are in the wrong place. The patient reader will learn in this book about incredible ideas that would be virtually unimaginable to humans had they not been demonstrated to us by nature itself. Accepted ideas of modern physics imply that it is possible to walk through walls, to make things that go backwards in time or even for which time stops, or liquids that flow forever, that there is no force of gravity, and that most of the energy of the universe is empty space. They do this in a way that is not only logically self-consistent but in agreement with our experiments and observations. Outlandish concepts are now routinely used to develop technologies that the lay person takes for granted – powerful microscopes, medical diagnostics and treatments, lightning fast communications, and so on. If you live in this world with no knowledge or understanding of the ideas behind such things, what distinguishes you from other animals?

This book grew out of a one-semester college course of the same name, aimed at non-science specialists with no math skills beyond (sometimes very rusty) arithmetic. Despite the fact that the subject matter involves great subtlety and in principle myriad unapproachable technicalities, I was surprised and delighted to learn after much trial and error that it could be done meaningfully. My reasons for writing the book were then three-fold: I could not find any one text that had the necessary but sufficient coverage, at a level of detail bearable by a tourist in science, while also treating its readers with the intellectual respect worthy of college-educated adults. More positively, I simply wished to share my enthusiasm for explaining the “big ideas” of modern physics with those whose might think comprehension impossible

with a limited background in science. Even though this book has been designed around my interaction with undergraduates in the arts, humanities, and business, it is also possible that other kinds of learner, including science specialists, might find the exposition useful in grasping the broad contemporary sweep of the field and an intuition to underpin calculation. My intention throughout has therefore been a direct, comprehensive but accessible survey of modern physics which requires no more (or less) from the reader than intellectual curiosity.

This is neither a popular science book nor a physics training manual. It is, unapologetically, a pure physics book and, to achieve this unity and clarity, there is only limited treatment of many important aspects where physics impinges upon other fields of science, technology, humanities, art and society, insofar as it illustrates the fundamental ideas. Unlike a typical physics textbook, a conscious effort was made to bring the reader to current ideas as quickly as possible and to resist the temptation to dwell on classical aspects, important as they still are as practical approximations at the human scale. At the other ends of the scale, this text deals only with fundamental ideas for which there is a consensus among physicists and for which considerable direct experimental evidence exists. Especially for a non-technical audience, I felt it important to distinguish this from speculation. (Of course, one cannot completely isolate modern physics from other fields or deny the continual theorizing that churns as part of the scientific process.) The aim is to present ideas in logical rather than chronological order – very often this is the same thing – without disturbing the flow too much with historical footnoting and detailed attribution, except where it enhances appreciation for the science. To compensate, a more comprehensive chronology of the milestones passed in modern physics, and the key scientists who achieved them, is given as a reference at the end.

The format of this printed book essentially surrenders to modern media (whatever that may be at the time you are reading this). Color, photos, animations, videos, interactive software, exercises, demonstrations, lab manuals, stuff not invented yet; all are done so much better, more topically, or at all, by modern media or in a classroom. Ideally, this textbook will therefore be read in conjunction with and as a portal to more

extensive reference material on the companion website or in the instructor's classroom, if there is one. However, given that traditional tools of mathematics beyond arithmetic are not to be used, the textbook compensates by squeezing the most out of language and simple pictures (sketches, diagrams, graphs). The reader will need to be able to comprehend quantitative data – arithmetic, units, powers of 10, simple graphs – at a level that is summarized early in the book. Why have a printed textbook at all? I believe use of a simple traditional book offers advantages for the goals of informing with understanding. Research has shown that learning takes place differently with screen-based reading; it is remembering versus understanding. Tactile reading matter, that allows one to physically flit back-and-forth, engenders a global appreciation for the subject and its interconnectedness. (Learners using this book would also, if at all possible, benefit from performing related laboratory work and hands-on activities – a range of suggested experiments can be found on the website – as another avenue to tactile interactive learning.) The ideas discussed in this book can and should be contemplated anywhere, anytime, so portability is a plus. Lastly, especially for students needing to grasp a very strange subject for the first time, the format is designed to limit, on the first pass, distractions from the main path, by requiring the reader to jump to a different medium if they want more bells and whistles. The ideas of modern physics are profound and subtle, requiring great *concentration*, even for physicists!

Chapter 1

Scientific Discovery

Science, the partisan of no country, but the beneficent patroness of all, has liberally opened a temple where all may meet.

Thomas Paine, *Letter to the Abbé Reynal*

Physics is a highly mathematical science at the root of all other sciences and the precursor of all technology (utility for living our lives). It seeks to make mental models of the physical world at its most fundamental level. From the point of view of a non-scientist, or even many scientists, it is an inaccessible subject requiring a certain mindset and years of study to penetrate. However, this view is false in so far as the *ideas* of physics – leaving aside the techniques that render those ideas *useful* - are communicable to anyone willing to make the effort to understand. Its techniques will be discussed in this book scarcely at all, but an appreciation of the origin of familiar modern technologies, examples of which are sprinkled throughout the chapters for illustration, can follow from comprehension of the underlying ideas. These ideas are often so radical and enlightening that they have impacted modes of thought far from the discipline. Thomas Paine, the best-selling author of the 18th century whose writings precipitated the

independence of the United States, was greatly influenced by the scientific revolution heralded by the transformative ideas of physicist Isaac Newton, for example.

To provide the proper context for the rest of this book, and to give the non-scientific reader a fighting chance of understanding the ideas of modern physics, before we come to physics, we must step back and ask, what is science? Most people would perhaps say that science is about making theories, then very carefully doing experiments to test them. While that is indeed the essence of the scientific process – sometimes it runs the other way when an unexpected experimental result suggests a new theory - it misses the key philosophy. In fact, non-scientists often project onto the discipline exactly the opposite of this key philosophy, thinking that the role of a scientist is proving a theory correct. However, this is impossible. The domain of a mathematician is logic and she may prove to you using logic that $1 + 1 = 2$. No doubt. No other possibility. This is what the word 'prove' means. But the domain of science is objective experiment and observation; this is how a scientific theory is validated. There are two reasons why science will never be able to prove anything with this method.

Experiments are performed in the messy real world of external influences, where equipment malfunctions, people get tired, things wobble, money runs out, etc. When an experiment is finished, no sane individual would claim that they could be absolutely certain of the result. Imagine the result of an experiment was a number, 6 say. But on Wednesday the apparatus could have wobbled a bit (this wasn't checked) making the recorded result higher than it should have been. So maybe the real result is 5; or 7 because wobbles can go both ways. In fact, scientists are usually not particularly preoccupied with the final number that the experiment produces. They usually have a pretty good idea what it will be at the outset, based on the proposed theory. Almost all their work actually goes into quantifying the uncertainty on the result. For example, carefully estimating the possible the effects of uncontrolled influences on the experiment, the result might be finally stated as 6 ± 1 with 90% confidence. This means that,

except for the influences one didn't control in the first experiment, if the same experiment were to be repeated again and again, 90% of the time one would expect to get a result between 5 and 7. Most of the effort in designing, building, and analyzing data in the experiment goes into estimating the " ± 1 with 90% confidence" and making it as small as possible by controlling as many of the influences as possible. With sufficient time, resources, and effort, one can make the degree of uncertainty on the final result as small as one wants. But it can never be made exactly zero, for one can never completely control every influence. There is always a carefully quantified element of doubt in the experimental result and, therefore, in the veracity of the theory against which one is comparing.

A second reason for in-built fallibility is that, while there are an infinite number of different ways one could test a theory by experiment, there are only so many that one could possibly do in practice. In consequence, even if a theory predicts a number that agrees to within 90% confidence of a certain experimental result, who can say whether it will continue to agree with a different kind of experiment that has yet to be performed?

So where does this leave us with the key philosophy of science? The philosopher Karl Popper noticed that many areas of human endeavor, not only in science, follow the method of making a theory which is then tested. The key philosophy he suggested, that distinguishes science from others, is that the testing is a bona-fide attempt to disprove a particular theory. Indeed, a theory can only be considered scientific if it is in principle possible to disprove it by performing an objective experiment or observation. Popper used the word *refutation*, which is more appropriate than disproof, since unavoidable uncertainty means that one cannot be perfectly sure that an idea has been ruled out either. (The uncertainties in science are usually so tiny that one can be extremely confident that a theory is wrong). The notion that science strives to show that theories are wrong rather than right, has gained general acceptance. It is an exceptionally economical and powerful approach to investigating the unknown that scientists try to keep in

mind, even though the day-to-day sociology of scientific practice and its appearance to outsiders may belie this. The power lies in the fact that while an infinite number of perfect experiments would be required for proof of any one theory, only a single reasonably careful experiment - more correctly, a series of identical independent experiments, performed by different scientists typically - is needed to refute a whole set of different theories. Settled scientific theory is found in textbooks, such as this one, but at the cutting edge of research there are usually many different competing models. Popper called them *conjectures* and the more of them one can quickly kill the better. The last theory standing after the bloodbath becomes the consensus, eventually leading to technology and exam questions for students.

It is interesting to apply this reasoning to theories in other disciplines as a test of whether the ideas themselves are scientific (try it). Some theories fail immediately because they are not even tested by objective experiment. For example, jazz is a distinctive style of music which could reasonably be said to correspond to a theory. The validity of this style of music is not tested objectively however. Along with a significant number of others, I detest jazz, although I am told it is liked by many people. The test of whether this theory is worthwhile is clearly subjective, not scientific. For other theories there may ostensibly be objective testing, but it is still not science according to Popper's definition. The example emphasized in his time was Psychoanalysis. From outward appearance this seemed like a scientific discipline with objective experiment and observation. The problem Popper had with the ideas, however, was that in his opinion the theory could accommodate any outcome of any psychological experiment. Whether or not it had some descriptive power, it was not in principle possible to refute the theory and therefore, by Popper's definition, not scientific. This does not make psychoanalysis any more wrong than it does jazz. People who pay money to experience either generally seem to come out the other end content, otherwise there would be no market for it.

The manner in which a consensus is reached about the correctness of major scientific theory is also perhaps surprising. Armed with our various conjectures about the explanation of the physical world, experiments are supposed to refute all options leaving (hopefully) just one. In practice one often finds that: a) the best consensus theory tends to remain so for a very long time (centuries) but does not survive un-refuted indefinitely; b) when the consensus eventually changes it does so quickly and dramatically. What this means is that our established ideas for the scientific description of nature are undergoing perpetual, if infrequent, revolution. Thomas Kuhn characterized this behavior as long periods of settled *paradigm* punctuated by short revolutionary bursts of paradigm shifting. A lot of the details and technological application of major scientific theories are worked upon during the stable paradigm. But at some point the uncertainty on experimental results becomes so small that they begin to disagree with the predictions of the current paradigm. Because science is a powerful force in our lives and scientists are only human, in the face of contradictory evidence there appears at first a cultural resistance to changing established ideas. This is not a bad thing since experiments have inherent uncertainties that are difficult to estimate. That is why it is so important in science to repeat the same experiment independently, to check whether some external influence was overlooked and not included in the estimated uncertainty. False trails appear far more often in science (and then in newspapers unfortunately) than genuine major advances. But if a result that refutes the established paradigm does not go away upon closer examination, a new theory is needed and for a short period, just a few years, there is pandemonium while candidates are put forward, refuted in turn, modified, until eventually a new paradigm emerges. Such machinations typically herald a new era of thought for the human race. This happened with the ideas of Isaac Newton three centuries ago. It also happened with the ideas of Relativity and Quantum Mechanics, which are the current paradigm for physics and the subject of this book.

It is noteworthy that when a paradigm shift comes along, the refuted theory that used to form the consensus is not discarded. Rather, it is simply recognized that it has a limited domain of

validity. It is still useful as an approximation to nature. It doesn't give quite the right result, but is often familiar, easy to work with, and precise enough for our technological purposes. (I do not discuss this kind much in the book, but scientific theories may have the domain of validity already built into them because they make approximations that are known, at the time of formulation, to break down at some level).

A good example of this evolution of scientific ideas is the notion of gravity. Everyone is familiar with the observation that terrestrial solid objects fall towards the surface of the Earth, unless something prevents them doing that. They also see apparently solid celestial objects like the Moon and the Sun move across the sky. In the 4th century B.C., the Greek philosopher Aristotle developed a paradigm for the explanation of these and other natural phenomena. It was not a scientific theory in the modern sense because the ideas were not subject to objective experimental testing, but we have to start somewhere! According to Aristotle, the motion of things is determined by their natural tendencies to move towards their proper place in the cosmos. Earthly things - earth and water - that one would call solids and liquids today, would therefore move towards the center of the Earth, which was naturally considered the center of the cosmos. On the other hand crystal spheres carried Heavenly bodies - the Sun, Moon and stars - to move eternally with unchanging circular motion about the Earth. The other planets in our solar system, which seemed like stars but had erratic motion, were carried by spheres within spheres, something so complicated it's difficult to imagine. The reign of Aristotelian ideas lasted for almost two millennia, at least in Europe, and provides the earliest known speculative theories of physics. Although better scientific ideas were later developed by scholars in Asia and even ancient Greece itself, they fizzled out along with the civilizations that sponsored them. Their lack of global impact and endurance was perhaps because they did not lead to accompanying technological revolutions. As is customary therefore, I will take up the story in mediaeval Europe, where discoveries in pure science would lead to world domination.

By the 16th century in Europe, astronomical measurements had improved to the extent that many of the features of the Aristotelian theory were becoming untenable. In particular, the paths of planets could no longer be accommodated by elaborate circles within circles around the Earth. Analyzing the data, Copernicus showed that the paths (orbits) appeared to go around the Sun, not the Earth, while Kepler determined their shapes to be elliptical. As I will discuss in more detail in chapter 2, as part of a comprehensive overhaul of our understanding of motion, in the late 17th century Isaac Newton proposed a new paradigm. Central to his theory was the concept of the *force* of gravity, transmitted through empty space and inexorably pulling all objects towards each other, deflecting them into curved paths. In particular, the Earth pulls everything towards its center including people, vehicles, houses, the Moon, the Sun, everything; and they all pull back on the Earth; and on each other. Newton was able to correctly calculate the motion of both terrestrial and celestial bodies from the same principle. In the space of a few decades, the universality of the principle of gravitational force and Newton's new mathematical methods for implementing the idea proved transformative in science – a new paradigm. Sorting out who pulls what, where, by how much, and exactly what the consequent movement is... well, that is part of the “techniques” of physics that this book assiduously tries to avoid. But understanding the underlying concepts is something anyone can aspire to.

The concept of gravity force governing the motions of all objects in the universe held sway for the next 200 years, until the beginning of the 20th century when astronomical observations had once more become sufficiently precise that small deviations of the results from theory could no longer be reasonably ignored. One famous discrepancy involved the movement of the planet Mercury's perihelion – the place on its orbit where it is closest to the Sun. As illustrated in figure 1.1, while Mercury takes only 88 days to move once around the Sun in its elliptical path, over a much longer period (thousands of years) the path itself has tended to slowly slide around. Astronomers were able to accurately measure this precession by telescope observations that tracked, over several decades, the small shift of the perihelion. Newton's gravity theory predicts

that Mercury's perihelion should move, but the number it gives is only 99% of the one observed, and the uncertainty on these observations was below 1% by the beginning of the 20th century. A new paradigm was needed again.

FIGURE 1.1

The point of closest approach in Mercury's elliptical orbit around the Sun shifts very slowly, compared to the speed of the planet in its orbit. The solid and dotted lines show the orbital path at widely different times.

Over a few decades in the early 20th century, Newton's theory of gravity was supplanted as the fundamental model by Einstein's General Relativity, which agrees with the observed movement of Mercury's perihelion to within current observational uncertainty. In fact, it was another, much more dramatic discrepancy in both theory and experiment concerning the speed of light that would eventually lead to Einstein's ideas taking prominence. But that is a more complicated story that will form a substantial part of this book later. The new paradigm, as I am trying to impress upon the reader in this chapter, was more than just a more precise technique for calculating. It ushered in many, many entirely new concepts of how one should think about physical reality. In particular, in General Relativity there is no gravity force as there was in Newton's model. Rather the movement of everything in the universe as a result of gravity is now seen as free motion in a space the fabric of which is itself curved, only giving the illusion of the presence of a force deflecting matter from its natural straight path.

Even though it is no longer considered quite correct, teachers still present the techniques of Newton's theory of gravity, not Einstein's, to all except perhaps physics graduate students. It is an excellent example of a formerly pre-eminent theory that has now become an indispensable approximate description of the physical world, much easier to work with than the fiendishly difficult mathematics of General Relativity and sufficiently precise for most applications. In fact,

in colloquial speech we still make use of the ancient Greek viewpoint, admiring the Sun “rising” even though we know the Sun is doing no such thing - it’s the Earth rotating dummy! Moreover, while Relativity has withstood many tests to refute it, no scientist would consider it an absolute truth. Some day it is possible that an experiment will find a discrepancy that cannot be attributed to uncertainty, which persists when the experiment is repeated independently, and then the evolution of scientific ideas will continue.

In summary:

- Scientific theories are validated by objective experimental tests.
- Every experimental result has inherent uncertainty which must be quantified.
- Experiments set out to refute, not prove, scientific theories.
- Established theories are infrequently but, it seems, inevitably replaced by better ones.

1.1 Numbers in Science

As a preamble to the rest of the book, some mathematical rules of engagement are required. The following briefly summarizes how numbers are used in science, at the level you will need. Normally, straightforward technical material like this would be relegated to the Appendix of a book. However, for this book, it seems more appropriate to include it as part of the main narrative, since the ideas of modern physics question the basis for using numbers in science. Skipping or skimming too quickly this part will make subsequent chapters much harder to understand.

Units

Science deals with quantities that can be measured. The number that represents the result of the measurement, 10 meters, 12 seconds, or whatever it may be, must always be quoted with units (meters, seconds, etc.) in order for the number to have any meaning. The very essence of measurement is a comparison of something with a standard unit. If I announce that the result of my measurement of the width of my house is 25, nobody knows whether I have a big house 25 meters wide or a small house 25 feet wide. Worse still, if I just announced that I made a measurement and the result is 25, it is not even clear what I am measuring. To put it another way, if I measure the width of my house the number I get will be different depending on whether I choose to measure in meters or feet, even though the width is the same in both cases. So it is crucial for giving meaning to any number in science that one specifies the units that go with it. The units that accompany any number representing a physical quantity are usually abbreviated. 10 meters becomes 10 m. 12 seconds becomes 12 s.

Equations

Equations in physics are used either to define something, usually in order to provide a *description* of an observation, or to make a quantitative connection between already-defined things, usually in order to provide an *explanation* of what is observed. Both the following illustrations are used in this book.

The following equation defines what is meant by the word SPEED

$$\text{SPEED} = \text{DISTANCE divided by TIME}$$

In other words, if one takes the number for a particular distance, for example the width of my house, 25 m say, and divide by the time it takes me to walk from one side to the other, 25 s say, then $25 \text{ m}/25 \text{ s} = 1 \text{ m/s}$ (one meter per second) is by definition the speed at which I walk. Notice

how the numbers in the previous sentence cancelled out but not the units. Units for numbers corresponding to different kinds of quantity persist in an equation and actually go on to tell us the units for the answer. Notice also how units were not specified in the SPEED equation itself. The equation is true whatever units one uses for DISTANCE and TIME but, having chosen what units one will use for these, the units for SPEED are then also fixed – meters per second, or feet per minute, or furlongs per fortnight, etc. Lastly, notice that everyone can agree on this definition of speed because it is based on objective measurements that anyone can make. These are called *operational definitions*. (An example of a definition which is not operational would be something like: a “classic movie” is any film older than 20 years in which all the lead actors put in a great performance. Clearly not everyone would agree on whether performances were great or how that could even be measured objectively.)

The following equation connects the ENERGY and FREQUENCY of a quantum of light

$$\text{ENERGY} = \text{PLANCK times FREQUENCY}$$

Both ENERGY and FREQUENCY are defined in a much more general way by other equations. You need not know at this stage what they mean. But this equation draws a new connection between them in the case of a quantum of light. The quantity PLANCK represents a constant of nature – a certain number that never changes with place or time – that was introduced by Max Planck, who was the first to propose this equation and thus begin the new field of Quantum Mechanics. Equations that make a connection like this, rather than just defining some new quantity, may justifiably be called laws of physics because they have some explanatory power beyond mere description. They enable us to understand why the physical world behaves the way we describe it.

Symbols

Equations in science are usually written in symbolic form. For example the symbol D might be used to represent DISTANCE from one end of a house to the other. It doesn't represent the width of any house in particular but rather the idea of the width of a house. For any particular house the symbol D will assume a numerical value. For my house $D = 25$ m but for your house it will be a different number. Note the *italics* on the symbol. This is to emphasize that it is a variable quantity that will be a different numerical value in any given situation, even though the idea it represents is always the same. Indeed, such symbols are sometimes referred to as "variables". Symbols used in physics are usually taken from the English or Greek letters.

With symbols one can write word equations much more economically. If one defines (using equations) SPEED = S and TIME = T , then the word equation for SPEED is equivalently

$$S = D / T$$

In symbolic form an equation is often called a "formula", especially if it is considered important or fundamental. One can use a symbolic equation in any situation and in each case the symbols will assume a particular numerical value. If I run from one end of my house to the other, it takes only 5 s of time instead of the 25 s it took when walking. The equation will indicate that, in this situation, my speed was $S = 25 \text{ m} / 5 \text{ s} = 5 \text{ m/s}$.

An equation can be manipulated and rearranged using the rules of algebra. This technique will not be used in this book but an understanding of what a symbolic equation means will help. The equation for S can be rearranged to read

$$\text{DISTANCE} = \text{SPEED times TIME}$$

$$D = S \times T$$

The symbolic equation is usually abbreviated even further to $D = ST$, with the multiplication understood implicitly whenever two symbols are adjacent. Being able to rearrange an equation like this enables one to use it in new ways. For example, if I go to your house and walk from one end to the other at my regular walking speed, $S = 1 \text{ m/s}$, while measuring the time T it takes me, the previous equation can be used to *calculate* the width D of your house without actually *measuring* it. Equations are used to calculate, instruments are used to measure.

Powers

These are an arithmetic notation. Example: $2^3 = 2 \times 2 \times 2 = 8$. Two to the power Three = Three lots of Two multiplied together = Eight. The general idea can be expressed in symbolic form. If symbol n represents one of the whole numbers (one of 1, 2, 3, 4, ... etc.), then by definition

$$D^n = D \times D \times D \times D \times \dots \times D \quad (n \text{ lots of } D \text{ multiplied})$$

In physics, D might represent the value of some DISTANCE. One can use the numerical example above but there had better be some units to make sense. If in a particular situation $D = 2 \text{ m}$, then $D^3 = 8 \text{ m}^3$ (Eight meters to the power three, or Eight meters cubed). Notice how a power on the symbol works both on the number in a particular situation (2 in this case) and the units (meters m in this case). With powers one can make new definitions. For example, a quantity represented by the symbol V defined by the equation

$$V = D^3$$

This equation actually defines the VOLUME V of a cube from the DISTANCE D along one edge.

Negative powers are also possible and have a very simple meaning. For example $2^{-3} = 1/8 = 0.125$. In other words the answer is 1 divided by the answer one gets without the minus sign in

the power. This idea becomes very important when one needs to write down extremely small numbers, as is discussed below.

One particular kind of numerical power is very important in physics, namely the powers of ten: $10^1 = 10$ (ten); $10^2 = 100$ (one hundred); $10^3 = 1000$ (one thousand); $10^4 = 10000$ (ten thousand); etc. Adding one to the power adds a zero to the end of the number, making it larger and larger obviously. At the time of writing the national debt of the United States, which like many other developed countries now and historically is highly indebted relative to its income, is more than 10^{13} \$ = \$10000000000000 (10 trillion dollars). So it's very important that citizens understand powers of 10! For negative powers the results get smaller and smaller: $10^{-1} = 1/10$ (one tenth) = 0.1 (zero point one); $10^{-2} = 1/100$ (one hundredth) = 0.01 (zero point zero one); $10^{-3} = 1/1000$ (one thousandth) = 0.001 (zero point zero zero one); etc.

Scientific Form & Unit Prefixes

Numbers encountered in science, especially in physics, are often very big or very small. This makes them both difficult to comprehend and difficult to work with. In order to make them more manageable there are a couple of standard tricks in the physicist's bag.

Firstly, it is standard practice to write them in "scientific form" using powers of 10. A couple of examples:

$$67800 = 6.78 \times 10^4 \quad 0.000054 = 5.4 \times 10^{-5}$$

The expression on the right in each equation is the scientific form of the number. In a sense the powers of 10 carry the largeness or smallness of the number and the part in front carries the precision of the number (through its decimal places). Such notation becomes essential when the number of zeros starts to get out of hand because the number is extremely large or small, as the

national debt example above shows. In physics the issue becomes acute: for example, the size of the universe is about 10^{26} m, while the smallest distances ever probed are smaller than 10^{-20} m.

The general convention for writing a number in scientific form is $a \times 10^p$, where symbol a represents a number greater than 1 but less than 10, while p represents any of the positive or negative whole numbers (except not usually 1), so, -4, -3, -2, 2, 3, 4,, etc.

Another trick that physicists use to make extreme numbers more user-friendly is to define new kinds of unit. Recalling that the actual number for a measured quantity will depend on the unit one chooses to use, it is perfectly legitimate to cook up a unit that makes all numbers quite reasonable in a particular situation. However, it would soon become quite confusing if everybody was inventing their own units – such as defining “1 Dalley” to be the unit of distance equal to the width of my house - so there is a generally agreed way to rescale just a few commonly used units by using prefixes. You are probably already familiar with unit prefixes. The centimeter is one hundredth of a meter and the kilometer is one thousand meters. In abbreviated form one writes

$$1 \text{ cm} = 10^{-2} \text{ m} \quad 1 \text{ km} = 10^3 \text{ m}$$

We all agree to use the meter unit m and then the c (centi) and k (kilo) are prefixes that rescale it when convenient. The width of a standard American page is 8.5 cm, the rail journey from Moscow to Vladivostock is a distance of 9,258 km, etc. The prefix c can be thought of as representing the number 10^{-2} and prefix k representing 10^3 . It's like sucking the largeness or smallness out of the number and putting it into the unit. A whole range of prefixes are used in physics but only those needed in the text will be introduced, as they arise. The most energetic particle accelerator in the world, the CERN Large Hadron Collider, is so powerful its energy needs to be expressed with the Tera (T) 10^{12} prefix. The current US national debt is in excess of

10 T\$ (ten Tera Dollars). The smallest machines in the world are approaching a nanometer (nm) 10^{-9} m in size, not much larger than an atom.