Measurement Error

by Tom Coan (adapted by Simon Dalley) Southern Methodist University

Why: Uncertainty is associated with the result of any measurement, so this lab will investigate the sources of uncertainty how it can be quantified.

What: Several independent length measurements of the same object will be made in order to calculate an average, and also calculate reaction times using a falling ruler.

How: We use a pipe, candle flame, falling ruler for length measurements.

Random Error and Uncertainty

Random error is the natural variation in multiple measurements where measured values may be scattered high and low compared to the true value and have no fixed or predictable pattern. Such random errors occur for various reasons, in particular because of our inability to completely isolate the experiment from (random) external influences.

When two measurements are done, typically you will have two different readings because of random errors. Neither will be the true value, but by taking multiple measurements one can take an average with the reasoning that the average measurement will be close to the true value. More measurements will yield an average closer to the true value because random errors will tend to cancel out in taking the average.

Random error affects the **precision** of a measurement, which one indicates by the number of **significant digits** given for the measurement. In case you don't remember: all digits (inclusive) between the leftmost non-zero digit and the rightmost digit are significant digits. For example, the following numbers each have 4 significant digits (shown in bold):

2,314 **2.314** **9009** **9.090** 0.00000**9009** **9.000**

The final result of any measurements should always be quoted with an **uncertainty,** which expresses the range over which single measurements vary due to random error. How does one calculate an uncertainty? The smallest it can be is the "instrument precision", which is the uncertainty on a single measurement due to the scale on the measuring device. For example, a ruler with 1mm divisions cannot be used to reliably measure a length smaller than 1mm, so in that case the instrument error is ± 1mm. There may be additional contributions to the uncertainty that do not become evident until an average of multiple measurements is taken. For example, the process used, the object being measured, or the person making the measurement can all lead to random error that increases the uncertainty.

After making N independent measurements, there is usually a scatter in the data, with a maximum and a minimum value. A simple way to estimate the total uncertainty is then (max - min) / $\sqrt{N}$.

(If this comes out smaller than the instrument precision, then the instrument precision should be used as the total uncertainty). So a final result would be written:

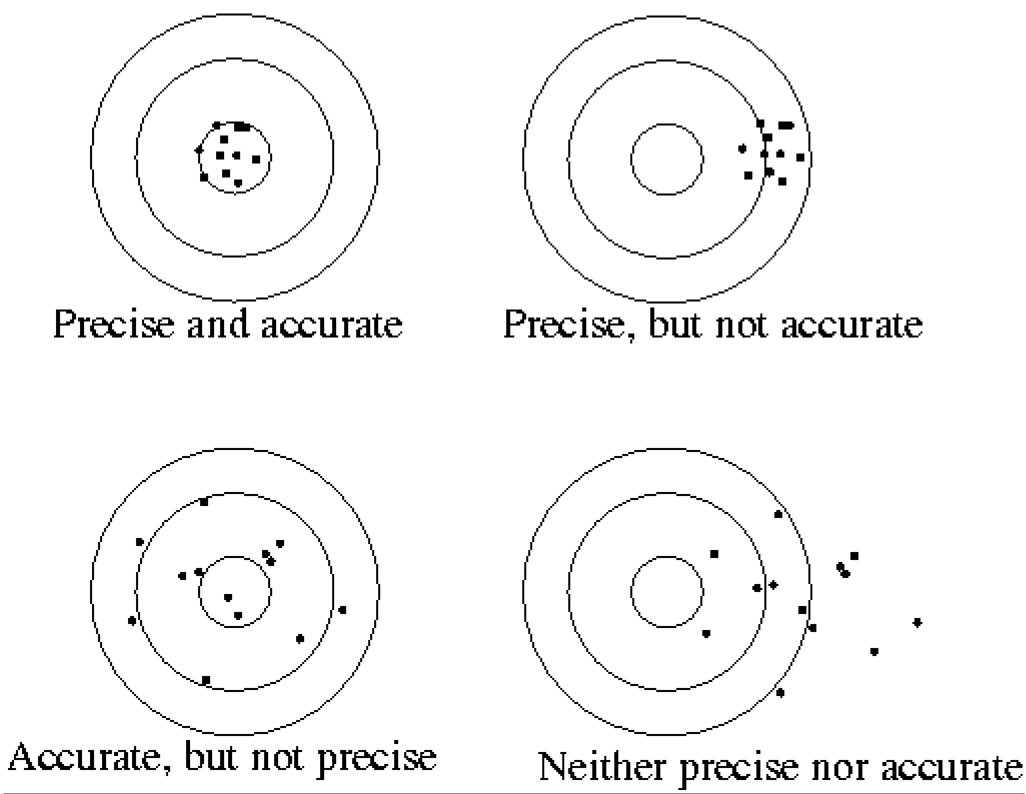
$$[\text{Average value}] \pm [(\text{max. value} - \text{min. value}) / \sqrt{N}\,]$$

- **As a guide, quote your uncertainty with one significant digit (round up as needed).**
- **Quote your average in the same units and with the same number of decimal places and power of 10 as your uncertainty.**

<u>Systematic Error</u>

Accuracy of measurement is the quality of nearness of the average to the true value. Sometimes you may also find that your measurements are *systematically* too high or too low compared to the true value, which affects the accuracy of your measurements, independent of their precision. For example, forgetting to add on the distance between the end of a ruler and the 0 point on its scale will yield length measurements that are inaccurate, always less than the true value. Systematic error is usually evident if you have some other source against which to compare your measurements (such as a textbook value, or the result from someone else's experiment, or the result using a different measuring apparatus).

The following pictures illustrate the distinction between random error (precision) and systematic error (accuracy)



Equipment & Procedure

1. Measure the length of a metal rod.

Get a “special ruler” marked SR, a “normal ruler” and one rod. Find the instrument precision of both rulers and write this down. Using the special ruler, measure the length of the rod. Rotate the rod in various ways and re-measure. Then have your partner take several measurements in the same way. Average all the values.

Next, use the normal ruler to repeat all the measurements again, and average the values.

For each kind of ruler, estimate the total uncertainty of your measurements

Rewrite your result (average ± uncertainty) to an appropriate number of decimal places and significant digits, including units.

2. Measure the height of a candle flame.

Light your candle and let the flame burn steadily for a minute or so. Holding the normal ruler a small distance away from the flame, measure the height of the flame (be careful not to burn the ruler). Each partner should make several measurements. Extinguish the flame when you are done.

Record all your measurements, find the total uncertainty and average value, and quote the final result to an appropriate number of significant digits and decimal places, with units.

3. Measure your reaction time.

Your reaction time is the time that passes between some external stimulus and your first action. A dropped ruler will be used. Have your partner hold the normal ruler vertically, holding it by the top and having the zero point toward the bottom. Place your thumb and forefinger at the ruler’s bottom, surrounding the zero point. Be prepared to pinch the ruler as if it were to fall. **Rest your forearm on the lab table to steady your hand**. Your partner will drop the ruler without warning. Pinch and grab the falling ruler as fast as you can; record how far it fell. Make 5 measurements like this. Repeat for your partner's reactions (who should record their own data). Each partner should find the maximum, minimum and average length the ruler fell for their reaction.

Reaction time *t* can be calculated using Galileo's formula for free fall:

$$t = \sqrt{(2s/g)}$$

where s = distance and $g = 9.8\ m/s^2$ is acceleration. Be careful to use the same distance units for s and g. Each partner should use their data for s to calculate their average reaction time and quote the result with an uncertainty. Whenever using a formula, always write the formula in your lab notebook and show how numbers are used in it for at least one example.

Conclusions

Write a paragraph summarizing the main things you learnt from **the data you collected and analyzed** in this lab. Quote actual data to support your conclusions.

Do not write about the procedure
Do not write about what you learnt how to do
Do not write about what you learnt from background reading

For example, you could discuss:

- Possible sources of error in the rod measurement. How well did your measurements with the special ruler agree with those done with the normal ruler? If there was a disagreement, what kind of error was it? Random or systematic? What caused this error?
- Possible sources of error in the candle measurement. What might you do to get a better measurement of the flame's height?
- Possible sources of error in the reaction measurement and their relevance.