# Moving Clocks 

By Tom Coan (adapted by Simon Dalley) Southern Methodist University
Based on the computer simulation program SPACETIME by Slavomir Tuleja http://stuleja.org/spacetime/

Why: We will investigate the effects of Special Relativity on time and speed.
What: We create computer images of clocks and observe how their time is affected by speed and acceleration, and how speeds add together in a Special Relativity

How: A computer simulation called SPACETIME is used to create clocks traveling at high speed in a velocity vs. position diagram and automatically calculates the effects on time.


The computer program SPACETIME depicts relativistic motion in one space dimension using two separate but related displays, as shown on the split-screen: the Space-time Highway and the more traditional Space-time (Minkowski) Diagram. On the highway, the plot shows position on the horizontal axis and velocity on the vertical axis, as if objects in different "lanes" are moving
at different speeds. The two windows share data, so you can view them independently or see both at once using the split screen. In this lab you will be using only the Space-time Highway.

## Procedure

Double click to open the folder SpaceTime-NEW on the desktop.
Feel free at any time to play with the program to see what the various commands do.
For reference, when the program starts there is a clock C 1 at position $\mathrm{x}=0$, velocity beta $=0$. Beta is the velocity as a fraction of light speed. By right-clicking on the screen you can create a new clock and by left-clicking you can drag a clock to a different position and velocity.

## Time Dilation

First you will investigate how time behaves when we take into account special relativity. Create 4 new clocks $\mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4, \mathrm{C} 5$ at $\mathrm{x}=0$ and give them the beta values of approximately 0.55 (clock C2), 0.7 (C3), $0.9(\mathrm{C} 4)$ and 0.98 (C5). You should see a total of 5 clocks on your screen, including the one (clock C 1 ) that is at rest (beta $=0$ ). Make certain you are viewing the

| Object | X position | $\beta$ | Clock time <br> (sec) | clock A time <br> clock time | $\gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0.00 | 0.00 | 5.0 | 1.00 |  |
| Clock__ |  |  |  |  | 1.00 |
| Clock__ |  |  |  |  |  |
| Clock__ |  |  |  |  |  |
| Clock_ |  |  |  |  |  |
|  |  |  |  |  |  | screen in the reference frame of clock

C1 (that's the one that should appear at rest) and advance the time on clock C1, using the up/down arrow keys, until it reads 3.00 seconds. Examine the times on all the other clocks and fill a data table like that shown here. You may find it useful to use the so-called Object Table on the right.

1. Plot the data on a graph of Gamma versus beta and join the data with a smooth curve. Gamma is the factor by which time dilates.
2. Write down a precise statement about the rate at which a moving clock ticks and the speed of the clock (this is Time Dilation)
3. What do you notice about the size of the different clocks? Be as precise as possible and relate it to the ideas of relativity (this is Lorentz Contraction).

Set all clocks back to zero time. Right-click to jump to the reference frame of clock C5, then advance the time on clock C 5 until it reads 3.0 seconds. Examine the Object Table.
4. Describe and explain the differences between the Object Table now and when you were in reference frame of clock C 1 .

## Adding Speeds

Now you will investigate how speeds add when we take into account special relativity. Create a New File, which will reset the picture so that C 1 is at $\mathrm{x}=0$, beta $=0$. Now create 4 clocks in succession, all at $x=0$, but each with a speed 0.5 c faster than the clock before it. To do this, start with clock C 1 with beta $=0$. Next, create clock C 2 with beta $=0.5$ also at $\mathrm{x}=0$. Then jump to the reference frame of clock C 2 and create a new clock C 3 at $\mathrm{x}=0$ that has a speed 0.5 c with respect to clock C2. Continue by transforming to clock C3's reference frame and creating a clock C 4 that has speed 0.5 c with respect to C3. Finally, transform to C 4 's reference frame and create a clock C 5 with beta $=0.5$ with respect to C 4 . Now transform back to the rest frame of C 1 , the original frame.
5. What are the speeds of clocks C3, C4, and C5 as seen from C1's frame?

What speeds might you have expected if you had naively added speeds together?

## Twin Paradox

Twins are 6orn on earth. One of them decides to travel at great speed to a distance star and then return home to earth. Her twin stays at home. When the traveling twin returns home, 6oth twins stand side by side and compare ages. According to Special Relativity, the traveled twin should be younger than the stay-at-home twin due to time dilation during their journey at high speed. However, 6y the first postulate of Special Relativity, the same laws of physics apply in any inertial reference frame. Therefore, who is to say who stayed still and who travelled? From the point of view of the star-6ound twin, they see the Earth twin zoom away and Cater return to meet them. So shouldn't the Earth twin suffer time dilation? But when the twins meet again, that would imply each is younger than the other - a paradox. How do their ages really compare upon reunion and what is the flaw in the reasoning?

Start with a fresh screen. C1 will represent the Earth-bound twin. Create a clock C2 at $\mathrm{x}=0$, beta $=0.9$ to represent the star-bound twin. We will pretend that it takes 1 s (according to C1) for C2 to get to the star, where it then instantaneously turns around and heads back to Earth, taking another 1 s (according to C1) to return. Step C1 forward by 1s, which is in the current rest frame (beta $=0$ ).
6. What time does C 2 now read? How do you explain this from C 1 's reference frame?

To make C 2 reverse direction, right-click on it, select program, and then drag C 2 to beta $=-0.9$. Step C1 forward another 1 s .
7. How do the times on C1 and C2 compare at the end of the journey? How do you explain this from C1's reference frame? Did anything unusual happen to the time readings on each clock as C 2 turned?

Now we will perform the same experiment as viewed from C2's reference frame. Step the clocks back to time 0 and right-click to jump to C2's reference frame (on the way out to the star). Step the clocks forward until the exact time that C 2 turns.
8. What time is showing on clocks C 1 and C 2 at this time? How do you explain this now you are in C2's reference frame?

We now need to jump from the inertial reference frame of C2 just before it turns (that's the one it's in on its way to the star) to the inertial frame just after it turns (that's the one it's in on its way back from the star.) Note: at the moment when C2 is actually turning, it is accelerating and therefore is no longer in an inertial reference frame. Although Special Relativity does not apply simply to measurements in that frame, nevertheless the SPACETIME computer program does correctly calculate the times that would show on the clocks just before and after the turn. At the time of the turn, right-click on clock C 2 shown after the turn and jump to the reference frame of that object.
9. When viewed from C2's frame in this way, how does the time showing on C1 compare just before and just after the turn?

Finish the journey by stepping the clocks forward until they meet again at the same position.
10. Are the final times on both clocks any different from what you found in question 7 from C1's frame of reference? Is there a paradox? If not, what happened to avert it? What about the experience of each twin was not symmetrical between them?

