Introduction Sheet 5

Understanding Functions. Curve Sketching

Rules to Remember:

To accurately plot a graph of a function $y = f(x)$, it is best to use a computer plotting program. For this sheet, we are only concerned with *sketching* the graphs of functions, to gain a visual appreciation of them. This means drawing roughly the correct shape, slope, etc. of the graph and labelling important points, such as places where the axes are crossed, the function diverges, etc.., as well as labelling axes properly (including physical units).

Ultimately, the best way to sketch a graph is to plug in a few values of x and work out the corresponding values of y. After marking roughly the coordinates (x, y) of these points, then smoothly join up the dots! As you gain experience, you will learn to choose fewer and more useful values of coordinates to get an idea of the shape of the graph.

Some common shapes of graphs of functions and tips to note:

$y = mx + c$:

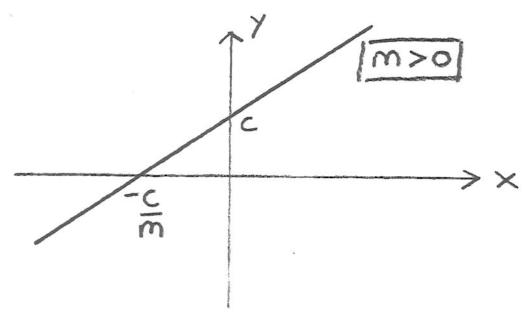
Straight line (linear)

m is the gradient (slope) of the graph

The gradient is constant

c is the intercept on the y axis (put $x = 0$)

$-c/m$ is the intercept on the x axis (put $y = 0$)



$y = ax^2 + bx + c$:

Quadratic

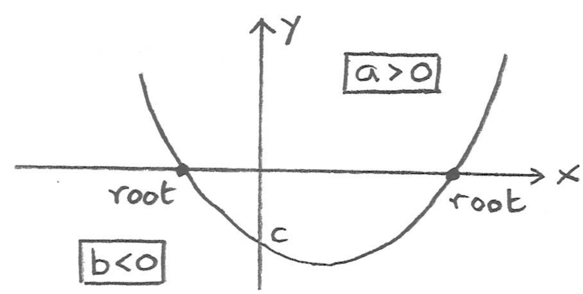
U-shaped if $a > 0$, upside-down-U if $a < 0$

The gradient changes with x

b is the gradient at $x = 0$

c is the intercept on the y axis

Intercepts with x-axis are roots of $ax^2 + bx + c = 0$



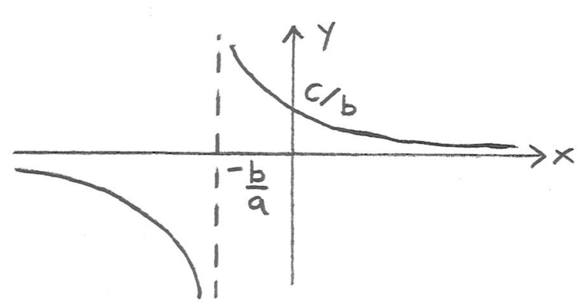
$y = \frac{c}{ax+b}$:

Falling to zero as $x \to \pm\infty$ $(1/\infty \to 0)$

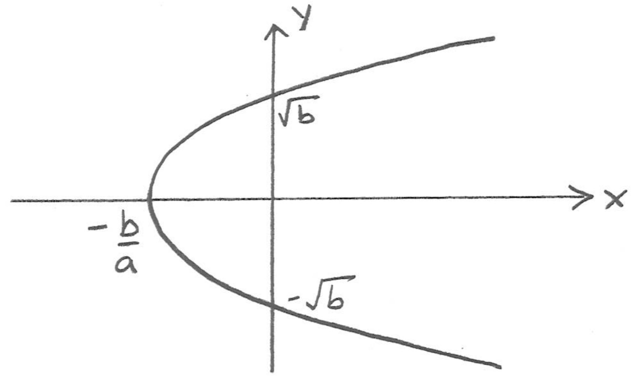
Diverges at $x = -b/a$

c/b is the intercept on the y axis

Never intercepts the x axis



$y = \pm\sqrt{ax+b}$:
$-b/a$ is the intercept on the x axis
$\pm\sqrt{b}$ are the intercepts on the y axis
Gradient is $\pm\infty$ at $x = -b/a$
Two $\pm$ solutions for $x > -b/a$
No solutions for $x < -b/a$
Shaped like a quadratic on its side
symmetrical about x axis



Practice Questions:
(you may wish to sketch with a pencil)

P1 Sketch the following linear functions, labelling the values of the x and y intercepts:

a) $y = x$ b) $y = 2x$ c) $y = 2x - 1$ d) $y = 3x + 2$
e) $y = -2x - 6$ f) $y = \frac{x}{3} + 4$ g) $2y = 4x + 8$ h) $3y + 3 = -x$

P2 Sketch the following quadratic functions, labelling the values of the x and y intercepts, if any:

a) $y = x^2$ b) $y = 2x^2$ c) $y = 4x^2 - x$ d) $y = 4x^2 - 1$
e) $y = x^2 + 4$ f) $y = -2x^2 + 4x$ g) $y = -4x^2 + 12x - 8$ h) $y = (x-2)^2$

P3 Sketch the following functions, labelling the values of the x and y intercepts and the values of x where y diverges:

a) $y = -\frac{1}{x}$ b) $y = \frac{1}{x-1}$ c) $y = \frac{2}{x+1}$ d) $y = \frac{3}{2x+3}$
e) $y = \pm\sqrt{x}$ f) $y = \pm\sqrt{4x}$ g) $y = \pm\sqrt{4x+9}$ h) $y = \pm\sqrt{16x-4}$

P4 When a diode allows a positive current to flow it has a resistance of $0.5\text{k}\Omega$. It does not allow negative current flow, i.e. the resistance is infinite. Using Ohm's law, sketch a graph of current I against voltage V for both positive and negative values. [Don't forget to label the physical units of your axes.]

P5 A ball is dropped from rest at a height of 20m above the ground. Assuming that the sum of kinetic and gravitational potential energy remains constant, sketch a graph of speed v against height h above the ground as the ball falls, labelling the value of the speed when the ball hits the ground. [Take the acceleration due to gravity $g = 10\text{ms}^{-2}$.]

P6 The pressure P, temperature T, and volume V of a perfect gas are related by $PV = nRT$, where n is the number of moles of the gas and R is the universal gas constant. If $nRT = 10\text{Nm}$, sketch a graph of P against V.