Light-Front Quantization of Conformally Gauge-Fixed Polyakov D1-Brane Action in presence of a Scalar Axion Field and an $U(1)$ Gauge Field [1]

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Abstract

Light-front quantization of the in the presence of a constant background scalar axion field $C(\tau, \sigma)$ and an $U(1)$ gauge field $A_\alpha(\tau, \sigma)$ is studied. The axion field C and the $U(1)$ gauge field A_α are seen to behave like the Wess-Zumino (WZ) fields and the term involving these fields is seen to behave like a WZ term for this action.

[1] **"Invited Talk" at the "International Conference on Light-Cone 2011: Relativistic Hadronic and Nuclear Physics", Southern Methodist University, Dallas Campus, Dallas, USA, May 22-27, 2011.**

The Polyakov action is one of the most widely studied topics in string theory [1]-[19]. Recently we have studied 9 - [16] the Hamiltonian [20, 21] and path integral [21, 22, 23, 24, 25] formulations of the conformally gauge-fixed Polyakov D1 brane action (CGFPD1BA) with and without a scalar dilaton field in the instant-form (IF) quantization (IFQ) [17, 18, 19, 20, 21, 22, 23, 24, 25] as well as in the light-front (LF) quantization (LFQ) [26, 27, 28, 29] (using the IF of dynamics on the hyperplanes defined by the Word-Sheet (WS) time $\sigma^0 = \tau = constant$ for IFQ and using the light-front (LF) dynamics on the hyperplanes of the LF defined by the light-cone (LC) time: $\sigma^+ = (\tau+\sigma) = constant$). In both the above cases the theory is seen (as expected) to be gauge anomalous and gauge-non-invariant(GNI) theory [9]-[16], possessing a set of second-class constraints in each case, owing to the conformal gauge-fixing of the theory. This describing a GNI theory (being a gauge-fixed theory) has been studied in Ref. [9] in the presence of a constant anti-symmetric 2-form gauge field $B_{\alpha\beta}(\equiv B_{\alpha\beta}(\tau,\sigma))$ which is a scalar field in the target space and an anti-symmetric field in the WZ space. This 2-form gauge field $B_{\alpha\beta}$ in this case is seen to behave like a Wess-Zumino(WZ) field and the term involving this field in the action is seen to behave like a WZ term for the CGFPD1BA. The CGFPD1BA (being a conformally gauge-fixed theory) is GNI and gauge anomalous and therefore it does not respect the usual (string) gauge symmetries defined by the WS reparametrization invariance (WSRI) and the Weyl invariance (WI) [1, 2, 3]. However, in the presence of a constant 2-form gauge field $B_{\alpha\beta}$ it is seen [9, 10] to describe a gauge-inavriant (GI) (and therefore gauge non-anomalous) theory respecting the usual (string) gauge symmetries defined by the WSRI and the WI.

In the present work we study this CGFPD1BA in the presence of a constant background scalar axion field $C(\equiv C(\tau,\sigma)))$ an $U(1)$ gauge field $A_\alpha(\equiv A_\alpha(\tau,\sigma)))$, where C is a constant scalar field in the target space as well as in the WS space and A_α is a scalar field in the target space and a vector field in the WS space. We find that the resulting theory obtained in the above manner describes a gauge-invariant system respecting the usual string gauge symmetries defined by the WSRI and the WI. It is seen that the axion field C and the $U(1)$ gauge field A_α, in the resulting theory behave like the WZ fields and the term involving these fields behaves like a WZ term for the CGFPD1BA. In the present work we study the LFQ of this GI theory describing the CGFPD1BA in the presence of the constant scalar axion field C and the $U(1)$ gauge field A_α under appropriate gauge-fixing conditions in the absence of boundary conditions. The Polyakov D1 brane action in a d-dimensional curved background $h_{\alpha\beta}$ is defined by [1, 2, 3]:

$$\tilde{S} = \int \tilde{\mathcal{L}} d^2\sigma \tag{1a}$$

$$\tilde{\mathcal{L}} = \left[-\frac{T}{2}\sqrt{-h}h^{\alpha\beta}G_{\alpha\beta}\right] \tag{1b}$$

$$h = \det(h_{\alpha\beta}) \tag{1c}$$

$$G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{1d}$$

$$\eta_{\mu\nu} = \text{diag}(-1,+1,...,+1) \tag{1e}$$

$$\mu,\nu = 0,1,....,(d-1)\ ; \qquad \alpha,\beta = 0,1 \tag{1f}$$

Here $\sigma^\alpha \equiv (\tau,\sigma)$ are the two parameters describing the worldsheet (WS). The overdots and primes would denote the derivatives with respect to τ and σ. T is the string tension. $G_{\alpha\beta}$ is the induced metric on the WS and $X^\mu(\tau,\sigma)$ are the maps of the WS into the d-

dimensional Minkowski space and describe the strings evolution in space-time [1, 2] (with d=10 for a superstring and d=26 for a bosonic string). $h_{\alpha\beta}$ are the auxiliary fields (which turn out to be proportional to the metric tensor $\eta_{\alpha\beta}$ of the two-dimensional surface swept out by the string). One can think of $\tilde{S}$ as the action describing d massless scalar fields X^μ in two dimensions moving on a curved background $h_{\alpha\beta}$. Also because the metric components $h_{\alpha\beta}$ are varied in Eq. (1), the 2-dimensional gravitational field $h_{\alpha\beta}$ is treated not as a given background field, but rather as an adjustable quantity coupled to the scalar fields [1,2,3]. The action $\tilde{S}$ has the well-known three local gauge symmetries given by the 2-dimensional WSRI and WI [1, 2, 3] as follows:

$$X^\mu \longrightarrow \tilde{X}^\mu = [X^\mu + \delta X^\mu] \tag{2a}$$
$$\delta X^\mu = [\zeta^\alpha(\partial_\alpha X^\mu)] \tag{2b}$$
$$h^{\alpha\beta} \longrightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] \tag{2c}$$
$$\delta h^{\alpha\beta} = \left[\zeta^\gamma \partial_\gamma h^{\alpha\beta} - \partial_\gamma \zeta^\alpha h^{\gamma\beta} - \partial_\gamma \zeta^\beta h^{\alpha\gamma}\right] \tag{2d}$$
$$h_{\alpha\beta} \longrightarrow [\Omega h_{\alpha\beta}] \tag{2e}$$

Where the WSRI is defined by the Eqs.(2a)-(2b) for the two parameters $\zeta^\alpha \equiv \zeta^\alpha(\tau,\sigma)$; and the WI is defined by Eq.(2c) and is specified by a function $\Omega \equiv \Omega(\tau,\sigma)$ [1, 2, 3]. Now for studying the so-called CGFPD1BA one makes use of the fact that the 2-dimensional metric $h_{\alpha\beta}$ is also specified by three independent functions as it is a symmetric 2×2 metric. one can therefore use these gauge symmetries of the theory to choose $h_{\alpha\beta}$ to be of a particular form [1, 2, 3]:

$$h_{\alpha\beta} := \eta_{\alpha\beta}; \quad h^{\alpha\beta} := \eta^{\alpha\beta} \tag{3}$$

For IFQ of the theory we study it on the hyperplanes defined by the WS time $\tau =$ constant, for which we take [1, 2, 3]:

$$h_{\alpha\beta} = \eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \tag{4a}$$
$$h^{\alpha\beta} = \eta^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & +1 \end{pmatrix} \tag{4b}$$
$$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta})} = +1 \tag{4c}$$

In the LFQ we study the theory on the hyperplanes defined by the LC-WS time $\sigma^+ := (\tau + \sigma) =$ constant, and use the LC variables defined by [1, 2, 3]:

$$\sigma^\pm := (\tau \pm \sigma) \quad \text{and} \quad X^\pm := (X^0 \pm X^1)/\sqrt{2} \tag{5}$$

In this case we take :

$$h_{\alpha\beta} := \eta_{\alpha\beta} = \begin{pmatrix} 0 & -1/2 \\ -1/2 & 0 \end{pmatrix} \tag{6a}$$
$$h^{\alpha\beta} := \eta^{\alpha\beta} = \begin{pmatrix} 0 & -2 \\ -2 & 0 \end{pmatrix} \tag{6b}$$
$$\sqrt{-h} = \sqrt{-\det(h_{\alpha\beta)}} = +1/2 \tag{6c}$$

Now the action $\tilde{S}$ in the so called conformal-gauge in both the instant- and front-forms of dynamics finally reads [1, 2, 3, 9, 10, 11, 12, 13, 14, 15, 16]:

$$S^N = \int \mathcal{L}^N d^2\sigma \tag{7a}$$
$$\mathcal{L}^N = [(-T/2)][\partial^\beta X^\mu \partial_\beta X_\mu] \tag{7b}$$
$$\beta = 0,1 \quad and \quad \mu = 0,1,i \ ; \quad i = 2,3,...,25 \ \ (IFQ) \tag{7c}$$
$$\beta = +,- \quad and \quad \mu = +,-,i \ ; \quad i = 2,3,...,25 \ \ (LFQ) \tag{7d}$$

The above action is the conformally gauge-fixed Polyakov D1 brane action. This action is seen to lack the local gauge symmetries. This theory in the IFQ is seen to be an unconstrained theory [12] and in the LFQ it is seen to possess a set of 26 second-class constraints [13, 14, 15, 16]. The Hamiltonian and path integral formulations of this CGFPD1BA in the IFQ have been studied by us in Ref. [12] and this action in the presence of a 2-form gauge field has ben studied by us in Ref. [9]. In this work, we study the LFQ of this CGFPD1BA in the presence of a constant scalar axion field C and an $U(1)$ guage-field A_α. The CGFPD1BA in the presence of a constant background scalar axion field C and an $U(1)$ gauge field A_α is defined by [1, 2, 3, 4, 5, 6, 7, 8]:

$$S^I = \int \mathcal{L}^I \ d^2\sigma \tag{8a}$$
$$\mathcal{L}^I = [\mathcal{L}^C + \mathcal{L}^A] \tag{8b}$$
$$\mathcal{L}^C = [\lambda \mathcal{L}^N] = \left[-\frac{T}{2}\right][\lambda \partial^\beta X^\mu \partial_\beta X_\mu] \tag{8c}$$
$$\mathcal{L}^A = \left[-\frac{T}{2}\right][-\Lambda C \varepsilon^{\alpha\beta} F_{\alpha\beta}] \tag{8d}$$
$$\lambda = \sqrt{(1+\Lambda^2)}; \quad \Lambda = constant; \quad \varepsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{8e}$$
$$F_{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha) \tag{8f}$$
$$f = F_{01} = -F_{01}(IFQ); f = F_{+-} = -F_{-+}(LFQ) \tag{8g}$$
$$\alpha, \beta = 0,1 \quad and \quad \mu = 0,1,i; \quad i = 2,3,....,25 \ \ (IFQ) \tag{8h}$$
$$\alpha, \beta = +,- \quad and \quad \mu = +,-,i; \quad i = 2,3,....,25 \ \ (LFQ) \tag{8i}$$

This action in the IFQ is seen to possess a set of 3 first-class constraints:

$$\Psi_1 = \Pi^0 \approx 0; \Psi_2 = (E - \Lambda TC) \approx 0 \quad and \quad \Psi_3 = \Pi_c \approx 0 \tag{9}$$

where $P^\mu, \Pi^0, E(\equiv \Pi^1)$ and Π_c are the canonical momenta conjugate respectively to X_μ, A_0, A_1 and C. In the LFQ, the theory is seen to possesses a set of six first-class constraints:

$$\chi_1 = [P^+ + (\frac{\lambda T}{2})(\partial_- X^+)] \approx 0; \quad \chi_2 = [P^- + (\frac{\lambda T}{2})(\partial_- X^-)] \approx 0 \tag{10a}$$
$$\chi_3 = [P_i + (\frac{\lambda T}{2})(\partial_- X^i)] \approx 0; \quad \chi_4 = \Pi^+ \approx 0 \tag{10b}$$
$$\chi_5 = (\Pi^- - \Lambda TC) \approx 0; \quad \chi_6 = \Pi_c \approx 0 \tag{10c}$$

Where P^+, P^-, P_i, Π_c, Π^+ and Π^- are the momenta canonically conjugate respectively to X^-, X^+, X_i, C, A^- and A^+. After including the primary constraints χ_i in the canonical Hamiltonian density $\mathcal{H}_2^c$ with the help of Lagrange multiplier fields $v_1(\sigma^+,\sigma^-)$ $v_2(\sigma^+,\sigma^-)$, $v_3(\sigma^+,\sigma^-)$, $v_4(\sigma^+,\sigma^-)$, $v_5(\sigma^+,\sigma^-)$ and $v_6(\sigma^+,\sigma^-)$ (which we treat as dynamical), the total Hamiltonian density $\mathcal{H}_2^T$ could be written as:

$$\mathcal{H}_2^T = [\mathcal{H}_2^c + v_1\chi_1 + v_2\chi_2 + v_3\chi_3 + v_4\chi_4 + v_5\chi_5 + v_6\chi_6] \tag{11}$$

The matrix of the Poisson brackets of the constraints χ_i is seen to be singular implying that the set of constraints χ_i is first-class [2]-[5] and that the theory described by S_2 is GI. The theory described by S_2 is, as before , again indeed seen to possess three local gauge symmetries given by the two-dimensional WSRI and the WI. The theory is indeed seen to describe a gauge-invariant system respecting the usual local string gauge symmetries: the WSRI and WI defined by [1, 2, 3]:

$$\begin{aligned}
X^\mu &\longrightarrow \tilde{X}^\mu = [X^\mu + \delta X^\mu] && \text{(12a)}\\
\delta X^\mu &= [\zeta^\alpha(\partial_\alpha X^\mu)] && \text{(12b)}\\
h^{\alpha\beta} &\longrightarrow \tilde{h}^{\alpha\beta} = [h^{\alpha\beta} + \delta h^{\alpha\beta}] && \text{(12c)}\\
\delta h^{\alpha\beta} &= \left[\zeta^\gamma\partial_\gamma h^{\alpha\beta} - \partial_\gamma\zeta^\alpha h^{\gamma\beta} - \partial_\gamma\zeta^\beta h^{\alpha\gamma}\right] && \text{(12d)}\\
A_\beta &\longrightarrow \tilde{A}_\beta = [A_\beta + \delta A_\beta] && \text{(12e)}\\
\delta A_\beta &= [\zeta^\alpha\partial_\alpha A_\beta] && \text{(12f)}\\
C &\longrightarrow \tilde{C}_\alpha = [C + \delta C] && \text{(12g)}\\
\delta C &= [\zeta^\alpha\partial_\alpha C] && \text{(12h)}\\
h_{\alpha\beta} &\longrightarrow [\Omega h_{\alpha\beta}] && \text{(12i)}
\end{aligned}$$

The theory could, however, now be quantized under appropriate guage-fixing for which we could choose, e.g., the gauge: $\theta = A^- \approx 0$. Finally, following the Dirac quantization procedure in the Hamiltonian formulation [20, 21] the nonvanishing equal LC WS time commutation relations of the theory could be obtained after a lengthy but rather straight forward calculation and are omitted here for the sake of brevity. In the path integral formulation, the transition to the quantum theory, is, however, made by writing the vacuum to vacuum transition amplitude called the generating functional $Z_2[J_i]$ of the theory in the presence of external sources J_i as follows [21, 22, 23, 24, 25, 28, 29]:

$$\begin{aligned}
Z_2[J_i] \;:=\; \int [d\mu] \exp\Big[i \int d\sigma^+ d\sigma^- \Big[& J_i\Phi^i + \Pi_c(\partial_+ C) + P^+(\partial_+ X^-) + P^-(\partial_+ X^+) \\
& + P_i(\partial_+ X^i) + \Pi^+(\partial_+ A^-) + \Pi^-(\partial_+ A^+) + p_{v_1}(\partial_+ v_1) + p_{v_2}(\partial_+ v_2) \\
& + p_{v_3}(\partial_+ v_3) + p_{v_4}(\partial_+ v_4) + p_{v_5}(\partial_+ v_5) + p_{v_6}(\partial_+ v_6) - \mathcal{H}_2^T \Big]\Big]
\end{aligned} \tag{13}$$

where the phase space variables of the theory defined by the action S_2 are $\Phi^i \equiv (C, X^+, X^-, X^i, A^+, A^-, v_1, v_2, v_3, v_4, v_5, v_6)$ with the corresponding respective canonical conjugate momenta: $\Pi_i \equiv (\Pi_C, P^-, P^+, P_i, p_{v_1}, p_{v_2}, p_{v_3}, p_{v_4}, p_{v_5}, p_{v_6})$. The functional measure $[d\mu]$ of the generating functional $Z_2[J_i]$ is obtained as:

$$
\begin{aligned}
[d\mu] \;=\; & (\Lambda T)[\lambda T\partial_-\delta(\sigma-\sigma')]^{3/2}[\delta(\sigma-\sigma')]^2[dC][dX^+][dX^-][dX^i][dA^+][dA^-] \\
& [dv_1][dv_2][dv_3][dv_4][dv_5][dv_6][d\Pi_c][dP^-][dP^+][dP_i][d\Pi^+][d\Pi^-] \\
& [dp_{v_1}][dp_{v_2}][dp_{v_3}][dp_{v_4}][dp_{v_5}][dp_{v_6}]\delta[(P^+ + \frac{\lambda T}{2}\partial_-X^+)\approx 0] \\
& \delta[(P^- + \frac{\lambda T}{2}\partial_-X^-)\approx 0]\delta[(P_i + \frac{\lambda T}{2}\partial_-X^i)\approx 0]\delta[(\Pi^+)\approx 0] \\
& \delta[(\Pi^- - \Lambda TC)\approx 0]\delta[(\Pi_c)\approx 0]\delta[(A^-)\approx 0] \qquad (14)
\end{aligned}
$$

The LFQ of the theory is now complete. The Polyakov D1 brane action in a d-dimensional courved background $h_{\alpha\beta}$ defined by $\tilde{S}$ is GI and it possesses the well-known three local gauge symmetries given by the two-dimensional WSRI and the WI. However, when we study this action under the conformal gauge-fixing to obtain the CGFPD1BA, we find that the CGFPD1BA given by S^N is no longer GI and it describes a gauge anomalous (and GNI) theory and it also does not possess the usual local string gauge symmetries being a gauge-fixed theory [10]. Hovever, this GNI theory when considered in the presence of a contant background scalar axiom field C and an $U(1)$ gauge field A_α is seen to become a GI theory possessing the three local string gauge symmetries defined by the two-dimentional WSRI and the WI [10]. The scalar axion field C and the $U(1)$ gauge field A_α are seen to behave like the WZ field and the term involving these fields is seen to behave like a WZ term for the CGFPD1BA [1, 2, 3], which in the absence of this term is seen to posess a set of second-class constraints and consequently describe a GNI theory which does not respect the local gauge symmetries defined by the WSRI and WI [1, 2, 3, 9, 10]. The situation in the present case, as pointed out in the foregoing, is analogous to a theory where one considerers the CGFPD1BA in the presence of the constant 2-form gauge field $B_{\alpha\beta}$, where $B_{\alpha\beta}$ which is a scalar in the target space and an antisymmetric tensor in the WS space, behaves like a WZ field [9, 10] and the term involving this field behaves like a WZ term for the CGFPD1BA [9, 10].

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