Confinement and Higher Fock States in Light-Front Holography

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Applications of Light-Cone Coordinates

to Highly Relativistic Systems

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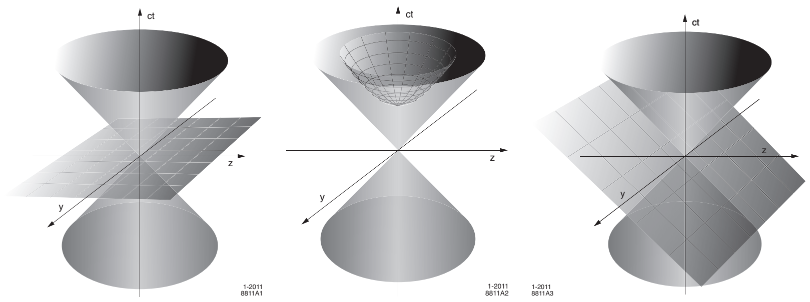


1 Introduction

Gauge Gravity Correspondence and Light-Front QCD

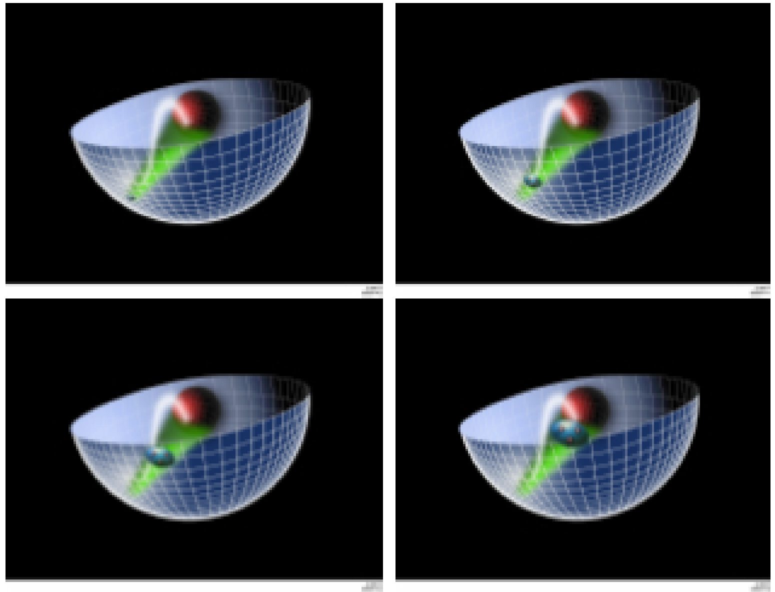
- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity in AdS space and conformal field theories in physical space-time have led to an analytical semiclassical approximation to light-front QCD, which provides physical insights into its non-perturbative dynamics
- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows to compute matrix elements diagonal in particle number
- Calculation of matrix elements $\langle P+q|J|P\rangle$ requires boosting the hadronic bound state from $|P\rangle$ to $|P+q\rangle$: boosts are trivial in LF
- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT methods possible
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS_5, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space (Dim isometry group of AdS_{d+1} is $(d+1)(d+2)/2$)

- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\left(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\right)$$

- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

$$L_{\rm Minkowski} \sim \frac{z}{R} \, L_{\rm AdS}$$



- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space

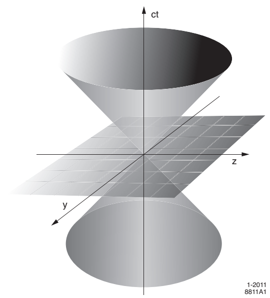
- Short distances $x_\mu x^\mu \to 0$ map to UV conformal AdS$_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ map to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z

- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

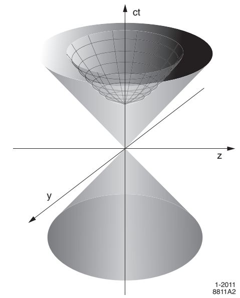
2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical *vs.* kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



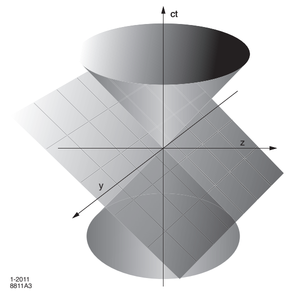
$$H,\ \mathbf{K}\ \text{ dynamical}, \qquad \mathbf{L},\ \mathbf{P}\ \text{ kinematical}$$

- *Point form*: hypersurface is an hyperboloid



$$P^\mu\ \text{ dynamical}, \qquad M^{\mu\nu}\ \text{ kinematical}$$

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$



$$P^-,\ L^x,\ L^y\ \text{ dynamical}, \qquad P^+,\ \mathbf{P}_\perp,\ L^z,\ \mathbf{K}\ \text{ kinematical}$$

Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from LF Lorentz invariant Hamiltonian equation for the relativistic bound state

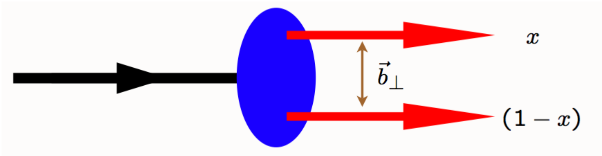
$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- Relevant variable in the limit of zero quark masses (dual to the invariant mass)

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iL^z \varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$ (P^+, $\mathbf{P}_\perp$, J_z commute with P^-)

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = |L^z|)$

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\left(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)}$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption

3 Light-Front Holographic Mapping of Wave Equations

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin, Vasiliev, Metsaev)
- Spin-J in AdS represented by totally symmetric rank J tensor field $\Phi_{M_1 \cdots M_J}$
- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\left(x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} \\ - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x,z)_{\mu_1 \cdots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin-J modes Φ_J with only physical degrees of freedom

  [Dosch, Brodsky and GdT (in preparation); Lyubovitskij *et al.*, operator construction (in preparation)]

- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \cdots A_J} = e^{M_1}_{A_1} e^{M_2}_{A_2} \cdots e^{M_J}_{A_J} \, \Phi_{M_1 M_2 \cdots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \cdots A_J}$$

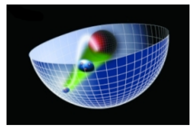
- Find effective action

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \partial_N \hat{\Phi}_{\mu_1 \cdots \mu_J} \partial_{N'} \hat{\Phi}_{\mu_1' \cdots \mu_J'} - \mu^2 \eta^{\mu_1 \mu_1'} \cdots \eta^{\mu_J \mu_J'} \hat{\Phi}_{\mu_1 \cdots \mu_J} \hat{\Phi}_{\mu_1' \cdots \mu_J'} \Big)$$

  upon μ-rescaling

- Variation of the action gives AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



  with $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$ and all indices along 3+1

Dual QCD Light-Front Wave Equation

$$\boxed{z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle}$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \to \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=\mathcal{M}^2\Phi_J(z)$$

find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$

with

$$U(\zeta)=\frac{1}{2}\varphi''(z)+\frac{1}{4}\varphi'(z)^2+\frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

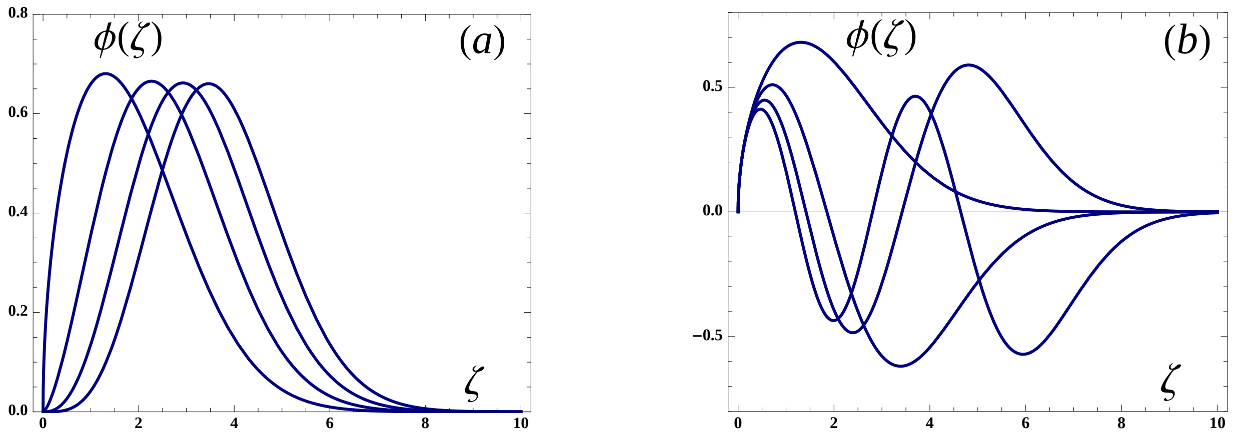
Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

- Choose Dilaton $\varphi(z) = +\kappa^2 z^2$ [$\varphi(z) = -\kappa^2 z^2$ does not provides area law for the Wilson loop]
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, |\phi(z)^2| = 1$

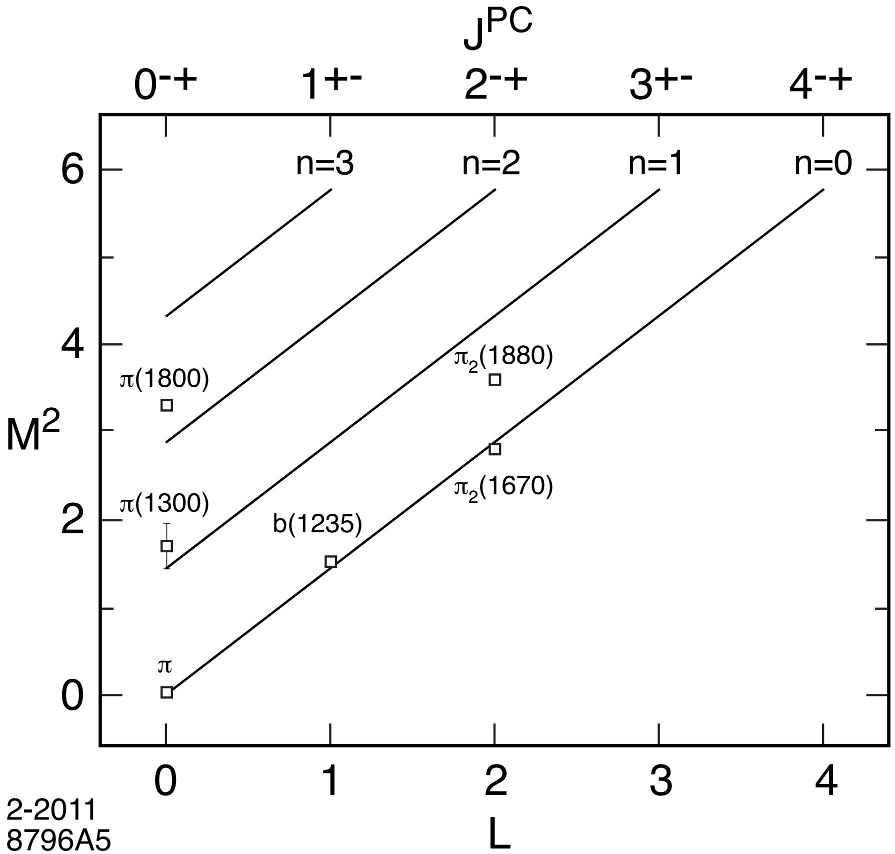
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

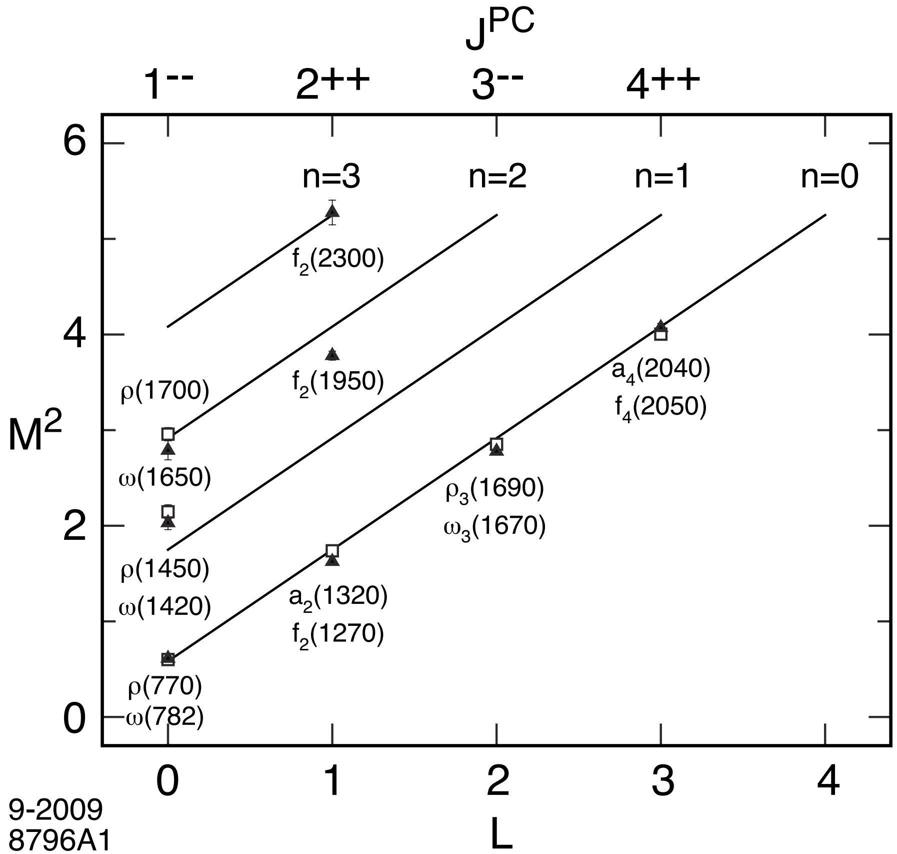
- Eigenvalues $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$



LFWFs $\phi_{n,L}(\zeta)$ in physical space time for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



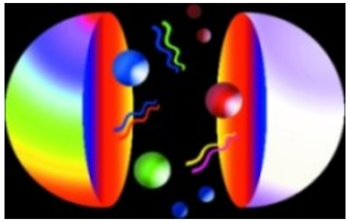


Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

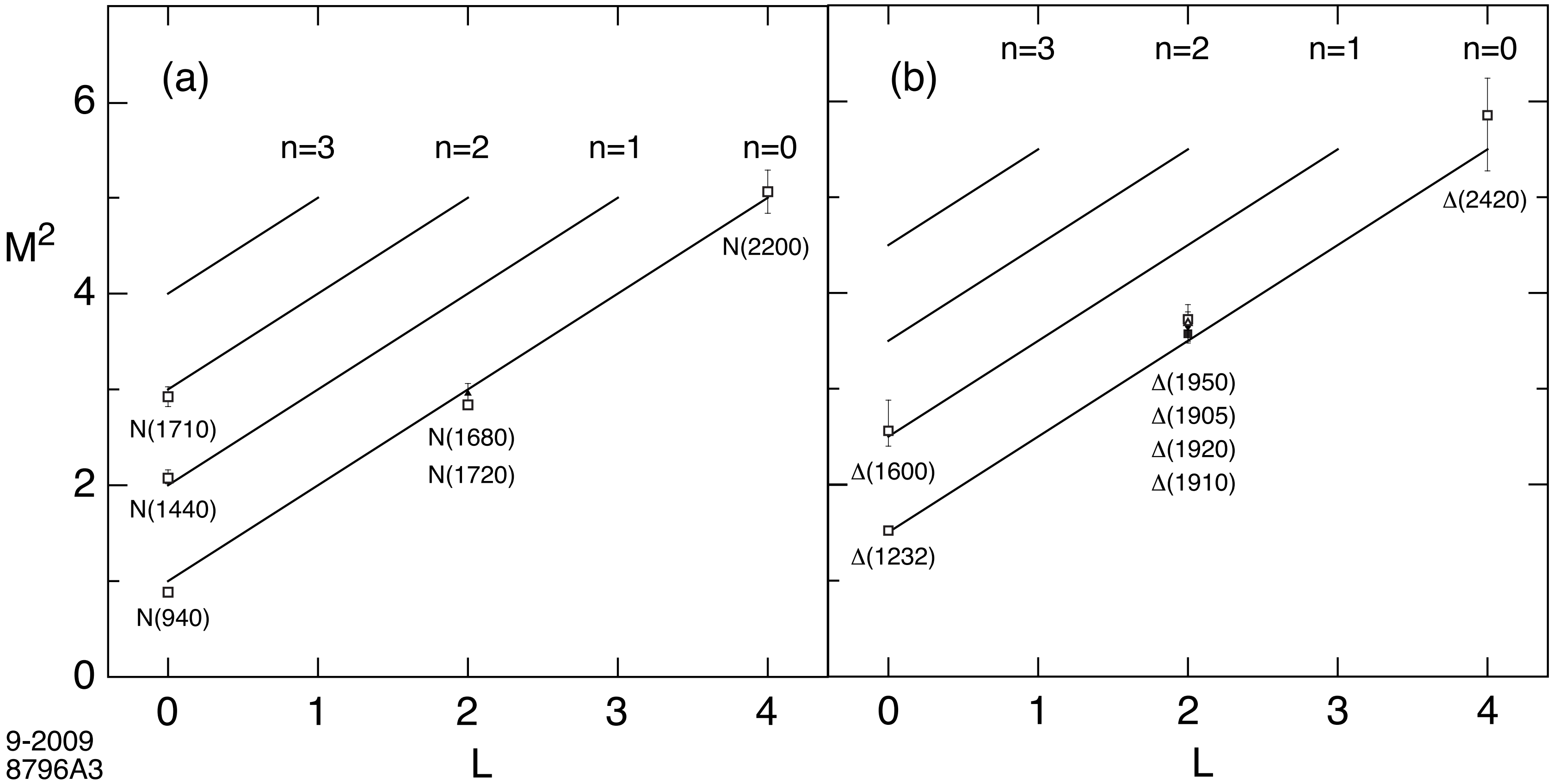
$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Same multiplicity of states for mesons and baryons!

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Regge trajectories for positive parity N and Δ baryon families ($\kappa = 0.5$ GeV)

4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006), PRD **77**, 056007 (2008)], PRD **78** 025032 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} A^M(x, z) \Phi^*_{P'}(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P \right) \epsilon_\mu \left(P + P' \right)^\mu F(Q^2)$$

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Mapping of J^+ elements at fixed light-front time: $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

- QCD Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\Big) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Consider a two-quark π^+ Fock state $|u\overline{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\overline{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q\!=\!0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx \, J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} \Phi(\zeta), \qquad z \to \zeta$$

- Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory (twist $\tau = N + L$) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

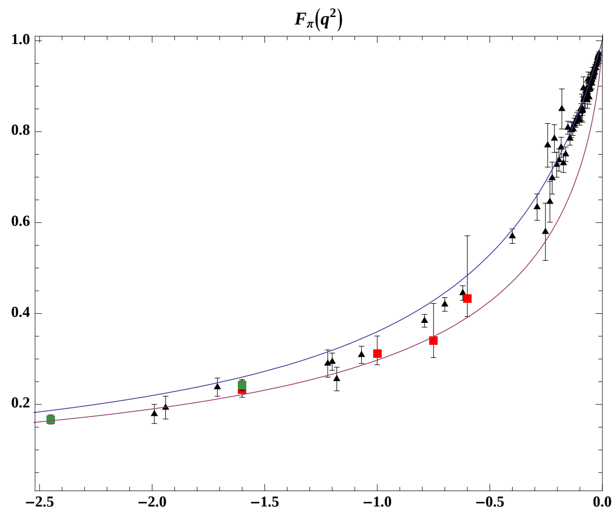
$$\boxed{F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{\tau-2}}^2}\right)}}$$

where ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$ [negative SL (dashed line) $\to$ positive TL dilaton (continuous)]

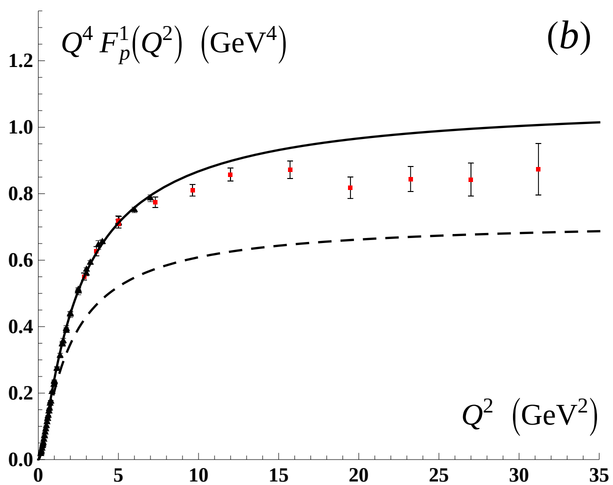
- Correct scaling incorporated in the model

- "Free current" $V(Q, z) = zQK_1(zQ) \to$ infinite radius (mauve), no pole structure in time-like region

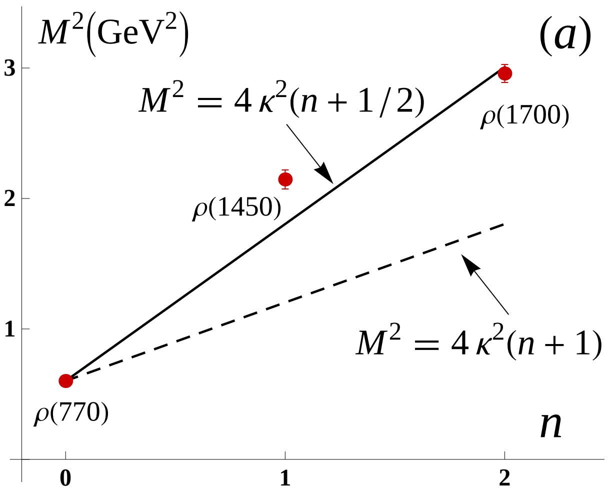
- "Dressed current" non-perturbative sum of an infinite number of terms $\to$ finite radius (blue)



Pion form factor ($\tau = 2$)



Proton form factor ($\tau = 3$)



VM radial trayectories

Nucleon Form Factors

- Light Front Holographic Approach [Brodsky and GdT]
- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode $\Psi_P(x,z)$

$$\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \overline{\Psi}_P(x,z)\, e^M_A \Gamma^A A_M(x,z) \Psi_P(x,z)$$

$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \langle \psi(P'), \sigma' | J^\mu | \psi(P), \sigma \rangle$$

- Effective AdS/QCD model: additional term in the 5-dim action

  [Abidin and Carlson, Phys. Rev. D79, 115003 (2009)]

$$\eta \int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \overline{\Psi}\, e^M_A e^N_B \left[\Gamma^A, \Gamma^B\right] F_{MN} \Psi$$

  Couplings η determined by static quantities

- Generalized Parton Distributions in gauge/gravity duals

  [Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

  [Nishio and Watari, arXiv:1105.290]

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

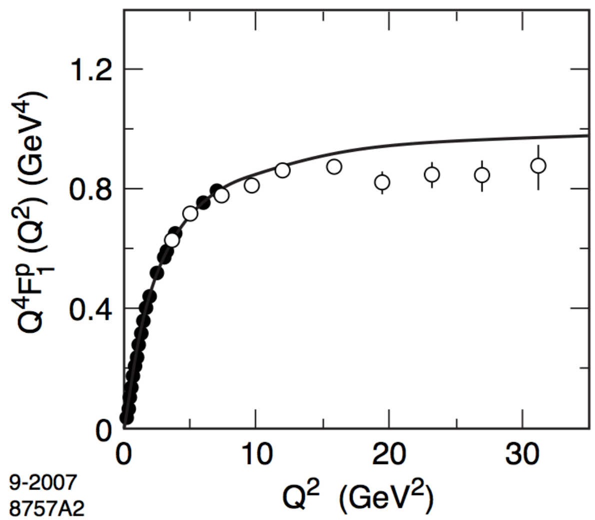
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\quad \Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_1{}^p_{N\to N^*}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

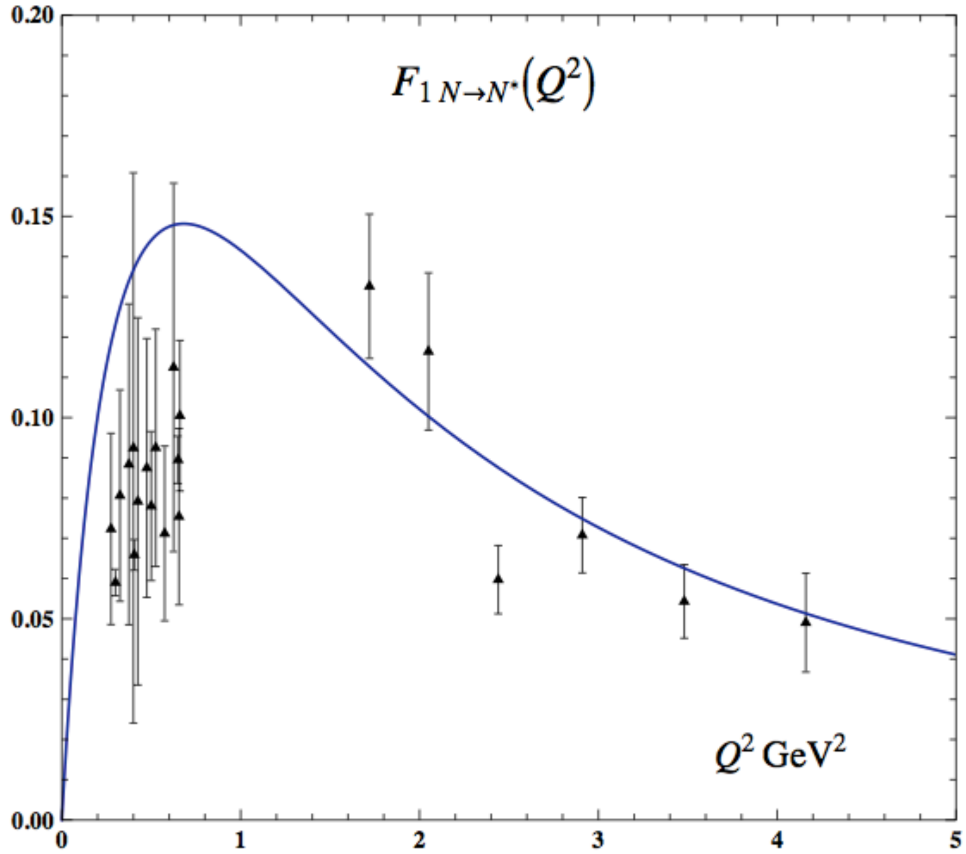
- Orthonormality of Laguerre functions $\quad \left(F_1{}^p_{N\to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n+1/2)$

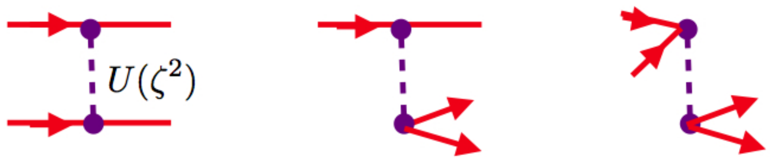


Data from I. Aznauryan, *et al.* CLAS (2009)

5 Confinement Interaction and Higher Fock States

[S. J. Brodsky and GdT (in progress)]

- Is the AdS/QCD confinement interaction responsible for quark pair creation?
- Only interaction in AdS/QCD is the confinement potential
- In QFT the resulting LF interaction is a 4-point effective interaction wich leads to $qq \to qq$, $q \to qq\overline{q}$, $q\overline{q} \to q\overline{q}$ and $\overline{q} \to \overline{q}q\overline{q}$



- Create Fock states with extra quark-antiquark pairs.
- No mixing with $q\overline{q}g$ Fock states (no dynamical gluons)
- Explain the dominance of quark interchange in large angle elastic scattering

  [C. White *et al.* Phys. Rev D **49**, 58 (1994)
- Effective confining potential can be considered as an instantaneous four-point interaction in LF time, similar to the instantaneous gluon exchange in LC gauge $A^+ = 0$. For example

$$P^-_{\rm confinement} \simeq \kappa^4 \int dx^- d^2\vec{x}_\perp \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\overline{\psi}\gamma^+ T^a \psi}{P^+}$$

Space and Time-Like Pion Form Factor

[GdT and S. J. Brodsky, arXiv:1010.1204]

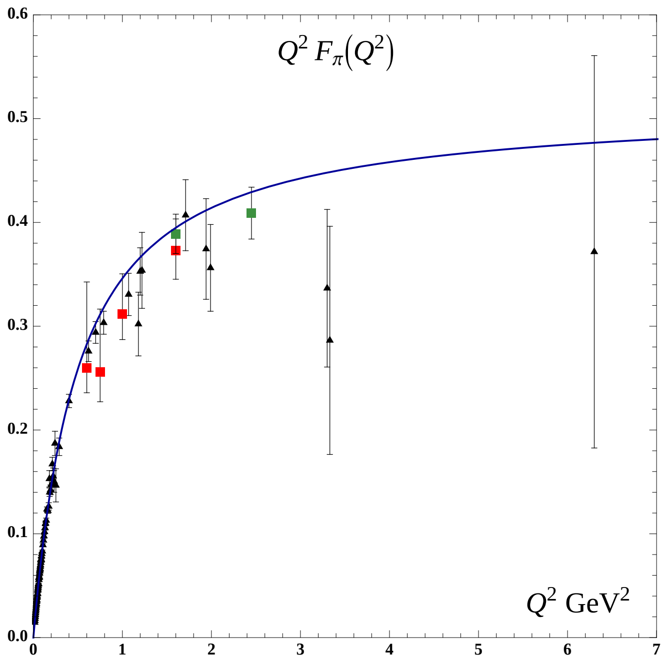
- Higher Fock components in pion LFWF

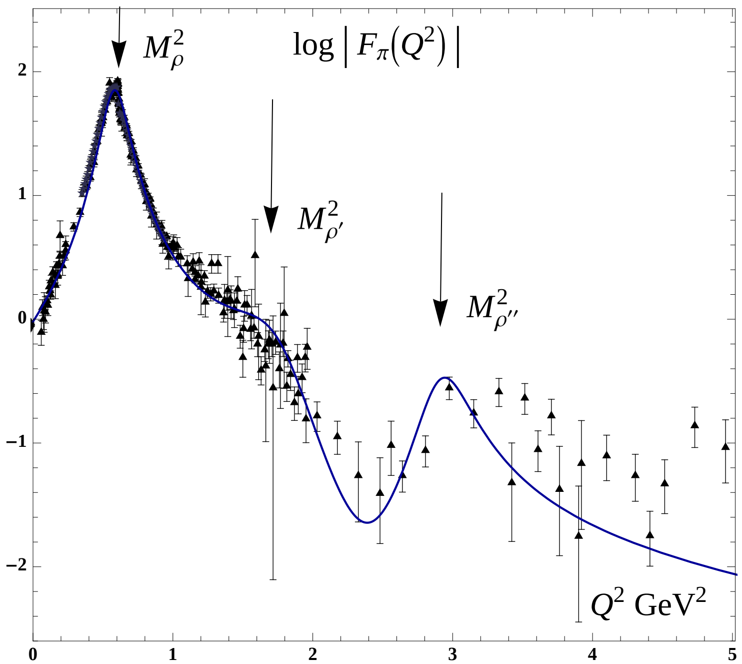
$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

corresponding to interpolating operators $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi$ and $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi\psi\overline{\psi}$

- Expansion of LFWF up to twist 4

$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130, \ \Gamma_{\rho'} = 400, \ \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$





Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$
$$\sim (2\pi)^4 \delta^{(4)}\left(p_\pi + q - k\right) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2P_{q\overline{q}}}\, \kappa\, z^2 e^{-\kappa^2 z^2/2}, \qquad \langle \Phi_\pi | \Phi_\pi \rangle = P_{q\overline{q}}$

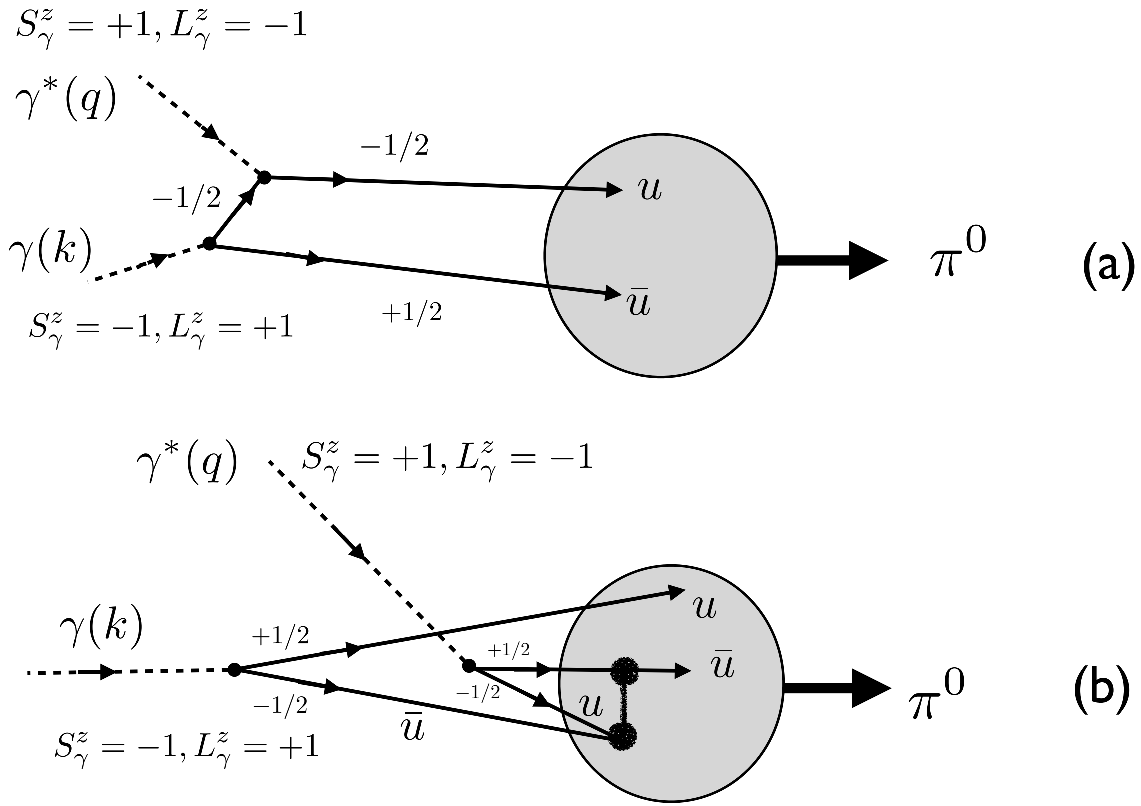
- Find $\quad \left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \sqrt{P_{q\overline{q}}}\, \kappa/\sqrt{2}\pi\right)$

$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{-P_{q\overline{q}} Q^2 (1-x)/4\pi^2 f_\pi^2\, x}\right]$$

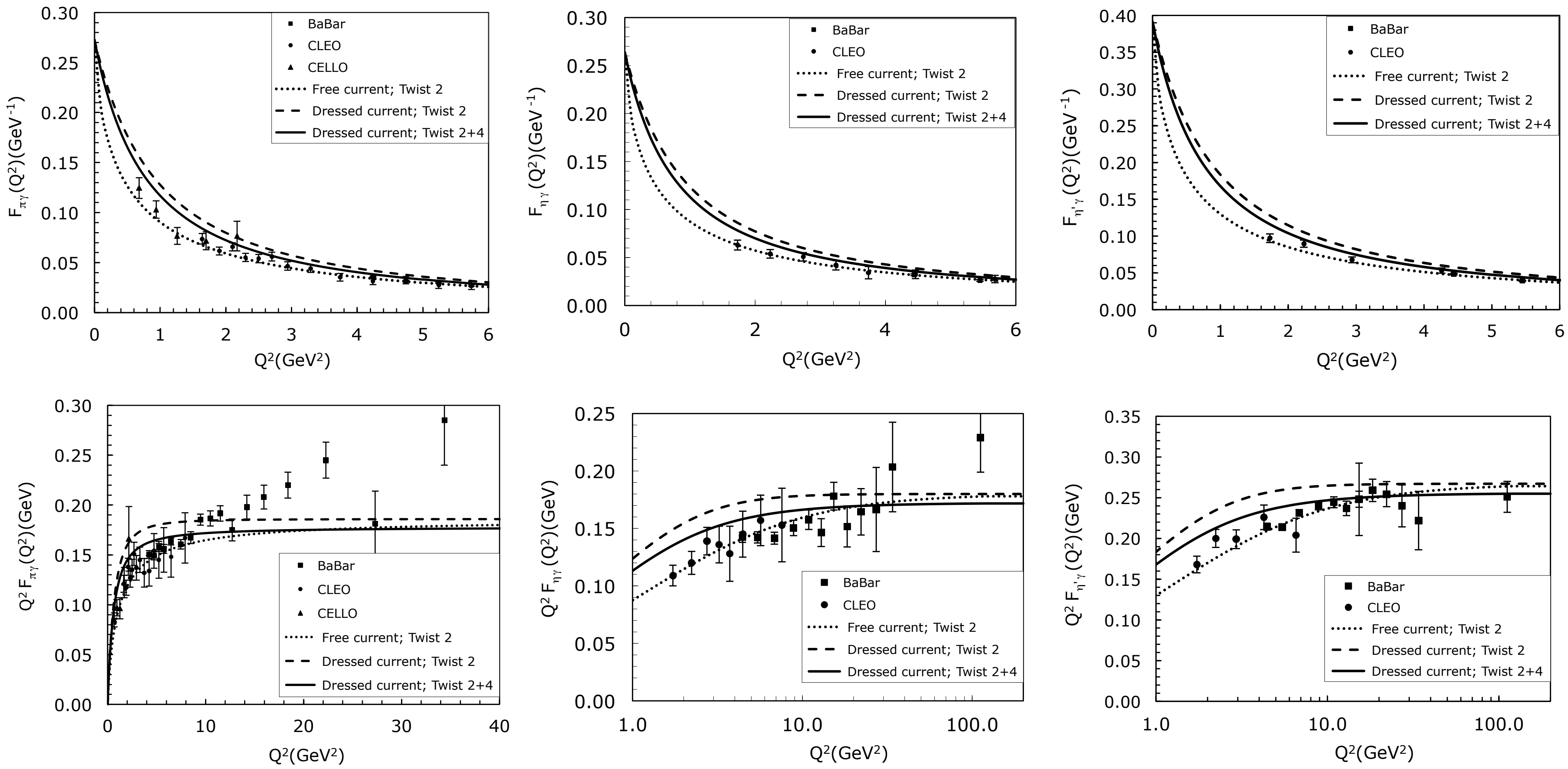
normalized to the asymptotic DA $\quad [P_{q\overline{q}} = 1 \to$ Musatov and Radyushkin (1997)]

- Large Q^2 TFF is identical to first principles asymptotic QCD result $\quad Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$

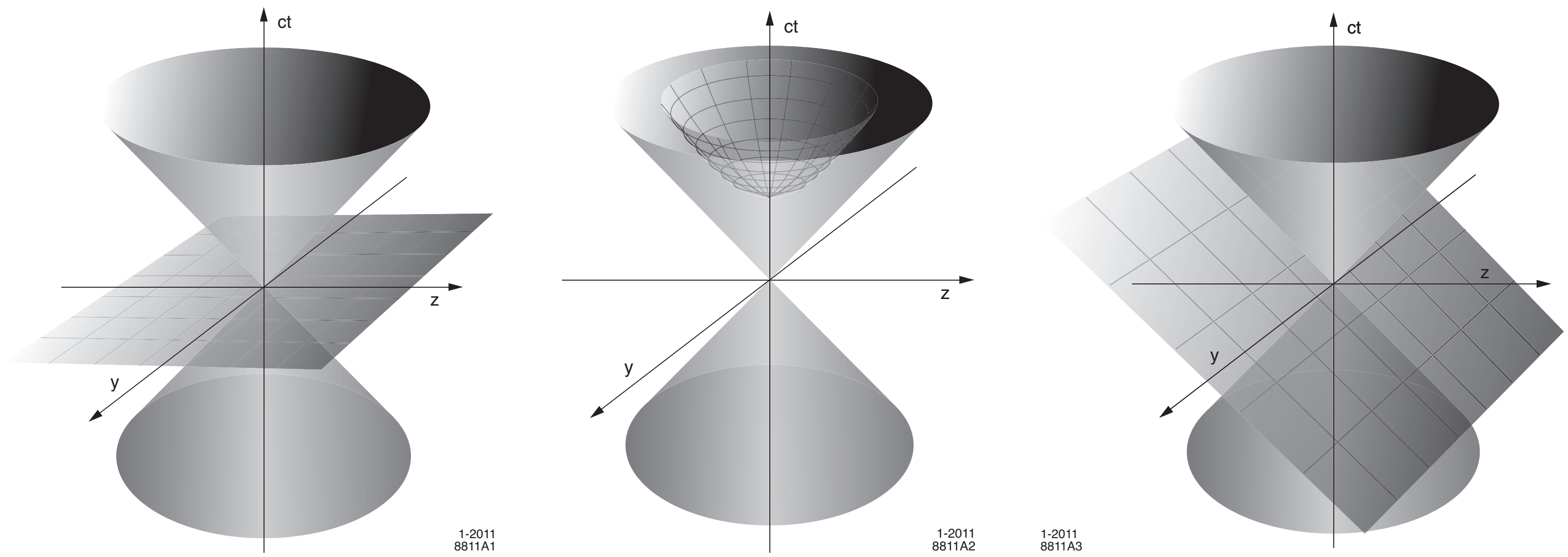
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA



Twist-two (a) and twist-four contribution (b) to $\gamma\gamma^* \to \pi^0$.



BaBar
CLEO
CELLO
Free current; Twist 2
Dressed current; Twist 2
Dressed current; Twist 2+4
$F_{\pi\gamma}(Q^2)(\mathrm{GeV}^{-1})$
$F_{\eta\gamma}(Q^2)(\mathrm{GeV}^{-1})$
$F_{\eta'\gamma}(Q^2)(\mathrm{GeV}^{-1})$
$Q^2 F_{\pi\gamma}(Q^2)(\mathrm{GeV})$
$Q^2 F_{\eta\gamma}(Q^2)(\mathrm{GeV})$
$Q^2 F_{\eta'\gamma}(Q^2)(\mathrm{GeV})$
$Q^2(\mathrm{GeV}^2)$



“ Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out ”

P.A.M. Dirac (1977)