Application of the systematic renormalization scheme in the covariant form of light-front dynamics to chiral perturbation theory

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Outline

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• Covariant Light-Front Dynamics
• Fock sector dependent renormalization scheme
• Taylor-Lagrange regularization scheme
• Application to ChPT
• Perspectives
Introduction

To understand the nucleon structure at low energy from a chiral effective Lagrangian we need an appropriate calculational scheme:

- relativistic
- non-perturbative
- well-controlled approximation scheme

We present LF$\chi$EFT – Light-Front Chiral Effective Field Theory
Light Front Chiral Effective Field Theory

Key points:

- **CLFD**
  - explicitly covariant formulation of light-front dynamics

- **FSDR**
  - Fock sector dependent renormalization scheme
  [V. Karmanov et al. PRD 77 (2008) 085028]

- **TLRS**
  - Taylor-Lagrange regularization scheme
Already done

**CLFD + FSDR + Pauli-Villars**

- Yukawa N=2, N=3
- QED N=2
  
  [V. Karmanov et al. PRD 77 (2008) 085028]
- ChPT N=2

**CLFD + FSDR + TLRS**

- Yukawa N=2
  

ChPT is the first “realistic” test of this approach
Covariant Light-Front Dynamics

Standard version of LFD

Rotational invariance is broken!

Covariant formulation

Arbitrary position of the LF plane

\[ \omega \cdot x = 0 \]
\[ \omega^2 = 0 \]

The state vector construction

Fock decomposition:

\[ \phi_{J\sigma}(p) \equiv |1\rangle + |2\rangle + \cdots + |n\rangle + \ldots \]

\[ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

\[ \phi_1 \quad \phi_2 \quad \phi_n \rightarrow n\text{-body light-front wave functions} \]

Truncation of the Fock decomposition: \( n \leq N \)

- \( N \) – the maximal number of Fock sectors under consideration
- \( n \) – number of constituents in a given Fock sector

Upper index \( \phi_n^{(N)} \): \( n \)-body light-front wave functions depend on the Fock space truncation

For example: \( \phi_2^{(2)} \neq \phi_2^{(3)} \)
**Vertex functions**

Wave functions $\leftrightarrow$ vertex functions:

$$\bar{u}(k_1) \Gamma^{(N)}_n u(p) = (s_n - M^2) \phi^{(N)}_n$$

$$s_n = (k_1 + \ldots + k_n)^2$$

Graphical representation:

Vertex function for a physical fermion made of a constituent fermion coupled to bosons
Vertex functions decomposition:

- invariant amplitudes constructed from the particle 4-momenta
- spin structures

Yukawa model:

\[ \bar{u}(k_1) \Gamma_1 u(p) = (m^2 - M^2)a_1 \bar{u}(k_1)u(p) \]
\[ \bar{u}(k_1) \Gamma_2 u(p) = \bar{u}(k_1) \left[ b_1 + b_2 \frac{M \phi}{\omega \cdot p} \right] u(p) \]
\[ \bar{u}(k_1) \Gamma_3 u(p) = \bar{u}(k_1) \left[ c_1 + c_2 \frac{M \phi}{\omega \cdot p} \right. \]
\[ \left. + C_{ps} \left( c_3 + c_4 \frac{M \phi}{\omega \cdot p} \right) \gamma^5 \right] u(p) \]

\[ C_{ps} = \frac{1}{M^2 \omega \cdot p} \epsilon^{\mu\nu\rho\lambda} k_2^\mu k_3^\nu p_\rho \omega^\lambda \]

\( a, b, c \) are scalar functions depending on dynamics
Renormalization scheme

Contribution to the physical fermion propagator

\[ \sum \delta m \rightarrow \delta m_2 \]

The general case: dependence on the Fock sector

\[ \Gamma_n^{(N)} \quad \delta m_{(N-n+1)} \quad (n-1) \text{ bosons} \]

(maximal number of particles in which the fermion line can fluctuate)

[V. Karmanov et al. PRD 77 (2008) 085028]
Renormalization scheme

Bare coupling constant: the same strategy

- Interaction with internal bosons

- Interaction with external bosons
Renormalization scheme

Iterative scheme: from sector to sector

- problem for $N=2$ $\Rightarrow$ $\delta m_2$, $g_{02}$
- problem for $N=3$ $\Rightarrow$ $\delta m_3$, $g_{03}$
- problem for $N=4$ $\Rightarrow$ $\delta m_4$, $g_{04}$
- $\vdots$
- $\vdots$
- $\Rightarrow$ $\delta m_N$, $g_{0N}$ for given $N$
Renormalization scheme

Renormalization conditions

- $\delta m$ is fixed from the solution of the system of equations for vertex functions

- Coupling constant is fixed from the condition on two-body components

\[
\begin{align*}
    b_1(s = M^2, x^*) &= g \\
    b_2(s = M^2, x^*) &= 0
\end{align*}
\]

$g$ is a physical coupling constant

In general, on-shell two-body components should not depend on $x$
Regularization

Infinite regularization schemes:

- Cut-off
- Dimensional regularization
- Pauli-Villars regularization scheme

All these schemes deal with infinitely large contributions

We use TLRS: systematic finite regularization scheme

Amplitudes depend on arbitrary finite scale
Basics of TLRS

\[ A = \int T(x)dx \rightarrow \int T(x)f(x)dx \]

\(f(x)\) – super regular test function

- \(f(x) = 1\) everywhere it is defined
- vanishes with all derivatives at boundaries

Support: \(f(x \geq H) = 0\)

Ordinary cut-off: \(H = H_0\)

We go beyond this ordinary cut-off
Basics of TLRS

\[ A = \int T(x)dx \rightarrow \int T(x)f(x)dx \]

Running boundary condition:

\[ H(x) = \eta x g_\alpha(x), \quad \eta > 1, \quad 0 < \alpha < 1 \]

Lagrange formula:

\[ f(ax) = -\int_0^\infty dt \partial_t f(xt) \]

When we put \( \alpha \to 1^- \):

- \( g_\alpha(x) \to 1 \)
- \( H(x_{\text{max}}) \to \infty \)
- \( f(x) \to 1 \) everywhere
- integration limit over \( t \) : \( xt \leq H(x) \)
  \[ t \leq \eta \]
Basics of TLRS: how it works

\[ \mathcal{A} = \int_0^\infty \frac{dx}{x + a} \rightarrow \int_0^\infty \frac{dx}{x + a} f(x) \]

New variable \( y \): \( x = ay \)

\[ \mathcal{A} = \int_0^\infty \frac{dy}{y + 1} f(ay) \]

New variable \( t \): \( z = yt \)

\[ \mathcal{A} = -\int_0^\infty \int_0^\infty \frac{dz}{z + a} \frac{dt}{z + t} f(z) \]

Integration domain: \( z = yt \leq H(y) \) \( t \leq \eta \)

\[ \mathcal{A} = \int_0^\infty dz \left( \frac{1}{z + a} - \frac{1}{z + \eta} \right) \] – Pauli-Villars type subtraction

Final result: \( \mathcal{A} = \ln \eta - \ln a \) \( \eta \) is an arbitrary finite positive number
Application to Chiral Perturbation Theory $N=2$

Need of a non-perturbative framework to calculate bound state properties

easy with $\pi NN$ coupling

to be generalized for $\pi\pi NN$ case
ChPT Lagrangian

- Lagrangian is formulated in terms of $u$ fields
  \[ u = e^{i \frac{\overline{\pi} \cdot \pi}{2F_0}} \]
  \( F_0 \) is the pion decay constant

- Expansion in a finite number of degrees of pion field
  \[ \mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots + \mathcal{L}^{(N)} + \ldots \]

- \( N \)-body Fock space truncation: \( 2(N - 1) \) pions
ChPT Lagrangian

In our first study

\[ \mathcal{L} = -\frac{g_A}{2F_0} \bar{\Psi} \gamma^\mu \gamma_5 \vec{\pi} \cdot \partial_\mu \vec{\pi} \Psi - \frac{1}{4F_0^2} \bar{\Psi} \vec{\pi} \cdot (\vec{\pi} \times \partial_\mu \vec{\pi}) \gamma^\mu \Psi \]

linear $\pi NN$ interaction

contact $\pi \pi NN$ interaction
Self-energy calculation

Attach test functions corresponding to internal propagators

\[ \Sigma \rightarrow \Sigma f\left(\frac{k_1^2}{\Lambda^2}\right) f\left(\frac{k_2^2}{\Lambda^2}\right) \]

Introduce a new variable \( t \) and apply the Lagrange formula

\[
\Sigma = \frac{3g_A^2 M}{32 F_0^2 \pi^2} \int_0^\infty dR_\perp \int_0^1 dx \int d\tau \frac{\mu^2}{R_\perp^2 + t^2(x^2 M^2 + \mu^2(1 - x))} f() f() \]
Self-energy calculation

Proceed in the common way:

- consider running boundary condition
- put $\alpha \rightarrow l^-, f^+ \rightarrow l$
- find integration limit $t \leq \eta$

The final result:

$$
\Sigma = \frac{3g_A^2 M \mu^2}{32 F_0^2 \pi^2} \ln \eta - \frac{3g_A^2 M \mu^2}{32 F_0^2 \pi^2} \int_0^1 dx \ln \frac{x^2 M^2 + \mu^2 (1 - x)}{M^2}
$$
Chiral limit

The nucleon mass correction from the self-energy contribution:

\[ M = M_0 + \Sigma \]

The full self-energy:

\[
\Sigma = \frac{3g_A^2 M \mu^2}{32F_0^2 \pi^2} \ln \eta + \frac{3g_A^2 \mu^2}{32F_0^2 \pi^2} \left( 2M - \frac{\mu^2}{M} \ln \frac{\mu}{M} - \frac{\mu \sqrt{4M^2 - \mu^2}}{M} \arctan \frac{\sqrt{4M^2 - \mu^2}}{\mu} \right)
\]

Non-analytic terms in the chiral limit:

\[
\Sigma = \frac{3g_A^2}{32F_0^2} \left( -\frac{\mu^3}{\pi} - \frac{\mu^4 \ln \frac{\mu}{M}}{\pi^2 M} + \frac{\mu^5}{8\pi M^2} + \ldots \right)
\]
Scalar form factor

To calculate it we proceed in a common way:
- attach test function
- introduce new variable \( t \)
- apply the Lagrange formula

\[
\sigma = \frac{3g_A^2\mu^2 M}{32 F_0^2 \pi^2} \int dR^2 d\alpha \int \frac{1}{R^2 + t^2 (x^2 M^2 + \mu^2 (1 - x))} f[f] \quad \rightarrow \quad \sim \Sigma \\
+ \frac{3g_A^2\mu^2 M}{32 F_0^2 \pi^2} \int dR^2 d\alpha \int \frac{t^2 \mu^2 (1 - x)}{[R^2 + t^2 (x^2 M^2 + \mu^2 (1 - x))]^2} f[f] \quad \rightarrow \quad \text{convergent integral}
\]

The final result

\[
\sigma = \frac{3g_A^2 M \mu^2}{32 F_0^2 \pi^2} \ln \eta + \frac{3g_A^2 \mu^2}{32 F_0 \pi^2} \left( 2M - \frac{2\mu^2}{M} \ln \frac{\mu}{M} - \frac{2\mu(3M^2 - \mu^2)}{M\sqrt{4M^2 - \mu^2}} \arctan \frac{\sqrt{4M^2 - \mu^2}}{\mu} \right)
\]

coinsides with the Feynman – Hell-Mann theorem: \( \sigma = \mu^2 \frac{\partial M}{\partial \mu^2} \)
System of equations

In the two-body Fock space truncation

\[ \bar{u}(p_1) \Gamma_1 u(p) = \bar{u}(p_1) (V_1 + V_2 + V_3 + V_4) u(p) \]
\[ \bar{u}(k_1) \Gamma_2 u(p) = \bar{u}(k_1) (V_5 + V_6 + V_7 + V_8) u(p) \]

The term \( V_8 \) corresponds to the contact \( \pi \pi NN \) interaction
Vertex functions representation

We choose the representation of vertex functions according to the vertex with an outgoing pion:

\[-i g_0 (k - \not\tau) \gamma_5\]

\[
\bar{u}(k_1) \Gamma_1 u(p) = \left( m^2 - M^2 \right) a_1 \bar{u}(k_1) u(p),
\]

\[
\bar{u}(k_1) \Gamma_2 u(p) = -i \bar{u}(k_1) \left( (k_2 - \not\tau) \gamma_5 b_1(R_\perp, x) + \gamma_5 \frac{M \not\tau}{\omega \cdot p} b_2(R_\perp, x) \right) u(p)
\]

\[
\tau = \frac{s - M^2}{2 \omega \cdot p}
\]

\[
s = (k_1 + k_2)^2
\]

With this representation \( b_1 \) and \( b_2 \) are just scalars.
Solution

\[
\begin{align*}
  b_1 &= \frac{2Ma_1g_0}{1 - \frac{f_0}{F_0^2} Z} \\
  b_2 &= 0 \\
  \delta m_0 &= -\frac{3g_0b_1}{4a_1 F_0^2} Z
\end{align*}
\]

\[Z \sim \Sigma\]

Condition : \( b_1 = g_A \)

Bare coupling constant : \( g_0 = \frac{g_A}{2Ma_1} \left(1 - \frac{f_0}{F_0^2} Z\right)\)

Mass counterterm : \( \delta m = -\frac{3g_A^2 M}{2F_0^2} \left(1 - \frac{f_0}{F_0^2} Z\right) Z \)
Final results

\[
\begin{align*}
    b_1 &= g_A \\
    b_2 &= 0 \\
    \delta m &= -\frac{3g_A^2 M}{2F_0^2} \left(1 - \frac{f_0}{F_0^2} Z\right) Z
\end{align*}
\]

Without contact interaction:

\[
f_0 = 0, \quad \delta m = -\frac{3g_A^2 M}{2F_0^2} Z = \Sigma
\]

With contact $\pi\piNN$ interaction:

\[
f_0 = 1, \quad \delta m = -\frac{3g_A^2 M}{2F_0^2} \left(1 - \frac{1}{F_0^2} Z\right) Z
\]
Nucleon mass corrections

First non-analytic contributions with and without contact interaction

\[ \delta m = \frac{3g_A^2}{32F_0^2} \left( -\frac{\mu^3}{\pi} - \frac{\mu^4 \ln \frac{\mu}{M}}{\pi^2 M} + \frac{\mu^5}{8\pi M^2} \right) \]

The first contribution of the contact interaction is of order \( \mathcal{O}(\mu^5) \)
Contributions to the nucleon mass

\( \delta m, \text{ MeV} \)

\( \mu, \text{ MeV} \)

- The leading non-analytic correction
- The full contribution of the nucleon self-energy
Perspectives

Calculations for ChPT $N=3$ (1 nucleon and 2 pions)