An Adjustable Parallel Plate Capacitor Instrument and Test of the Theoretical Capacitance Formula Obtained from Gauss's Law

Beau Wells*

Science Department, Allen High School,

300 Rivercrest Boulevard, Allen, TX 75002 USA

Emily Baker, Austin Farwell, Harrison Foster, Xiaohan Gao,

Benjamin Gruber, Erica Jones, Dennis Vu, and Sonya Xu

Jingbo Ye†

Physics Department, Southern Methodist University, Dallas, TX 75275 USA

(Dated: July 14, 2015)

Abstract

We describe an adjustable parallel plate capacitor apparatus designed for use in an undergraduate laboratory which permits precise variation of plate separation distances (10 µm increments) and overlap area. Two experiments are performed with the device to test the ideal capacitor formula derived from Gauss's Law. After correcting for edge effects and minor plate tilt, the device yielded capacitance values within 3% of theoretical values.

I. INTRODUCTION

There is a dearth of quality experiments that can be performed in an undergraduate laboratory to test classical electrostatics. Computer simulations help to fill this void, but there are few opportunities for students to conduct hands-on investigations to test fundamental principles like Gauss's Law. Parallel plate capacitors are routinely studied in introductory courses and would seem a simple lab tool, but they present problems because their capacitances are sensitive and can deviate substantially from theoretical values.

An idealized parallel plate capacitor, like those modeled in most textbooks, consists of two isolated conducting plates of zero thickness and very large (approximately infinite) surface area (A). When charged, the plates have perfectly uniform charge densities of equal magnitude (σ) and opposite sign, which create a uniform electric field between, and orthogonal to, the plates. From Gauss's Law, it can be shown that when the space between the plates is unfilled (or filled with air, since $\epsilon_r \approx 1$), this electric field has a magnitude of σ/ϵ_0, corresponding to a voltage between the plates of $\sigma d/\epsilon_0$, where d is the separation distance between the plates. Accordingly, the capacitance of the idealized capacitor is,

$$C_0 = \epsilon_0 \frac{A}{d}. \tag{1}$$

In practice, an ideal capacitor can only be approximated since, e.g., plate areas cannot be infinite (causing "edge effects"), plates cannot be made perfectly parallel (causing "tilt effects"), and plates cannot have zero thickness or otherwise have zero charge outside the inner face (resulting in additional "body capacitance"). Each limitation results in a variance between actual and idealized capacitance that can be quite large. These effects can be adequately controlled and estimated if a high precision capacitor is utilized, but such devices are not readily available in most laboratories. We describe such an apparatus, which is small, inexpensive, and easy to operate, and use it to conduct tests of the ideal capacitance formula.

II. DESCRIPTION OF OUR PARALLEL PLATE CAPACITOR APPARATUS

Our apparatus consists of two thin, square, parallel metal plates attached to blocks of insulating material. The plates have faces that are 10 cm × 10 cm and a thickness of roughly 1.5 mm. The top plate is firmly affixed to an insulating frame, while the bottom plate is attached to a smaller block of insulating material that is free to slide horizontally, allowing

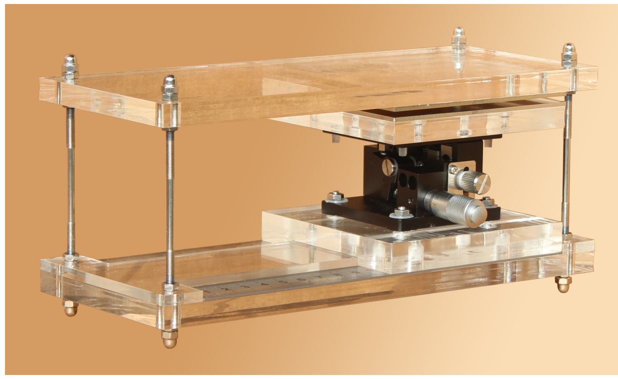

FIG. 1. The instrument.

the overlapping area of the plates to be adjusted by increments of 0.5 mm measured by a ruler along the bottom of the instrument. The plate overlap area can be adjusted from $0\,\text{cm}^2$ to $100\,\text{cm}^2$. The distance between the plates is changed using an adjustable micrometer head whose smallest unit of measurable change is 10 µm and which can be adjusted from 0 mm to 10 mm. While the plates should be perfectly parallel in theory, in reality the plates have a slight tilt that is too small to be visible. (As detailed below, our analysis provides a value for the average deviation in plate separation of roughly 30 µm.) The device permits insertion of a dielectric between the plates so that the property of such material can be studied.

III. ADJUSTING THE IDEAL CAPACITANCE FORMULA TO ACCOUNT FOR EDGE AND TILT EFFECTS AND BODY CAPACITANCE

A. Edge Effects

Edge effects result because the charge distribution and electric field of finite plates do not match those of infinite sheets of charge. Actual capacitor plates have non-uniform charge densities with charge more concentrated near edges and corners. Further, the electric field fringes near the edges of the plates and is lesser between the plates than for a corresponding ideal capacitor.[1,2] As a consequence, actual capacitance is larger than idealized. This effect is usually represented as a multiplier or ratio of actual to ideal capacitance, $\alpha = C/C_0$. This ratio depends on the dimensionless "aspect ratio", which for a square capacitor is defined as the ratio of the plate separation distance to the length of the side of a square plate ($b = d/L$).[3]

Several studies have investigated edge effects theoretically and empirically.[3–8] Nishiyama and Nakamura provide simple theoretical formulas for calculating edge effects for capacitors of various shape over stated ranges of aspect ratios.[3] Their work reveals that edge effects for square and disk capacitors tend to be similar and converge as aspect ratios become small (e.g., at an aspect ratio of b = .1, $\alpha_{disk} = 1.31809$ while $\alpha_{square} = 1.29980$). The aspect ratios in our study are very small ($.002 \leq b \leq .016$) and most closely overlap with the range ($.005 \leq b \leq .1$) applicable to the following disk capacitor edge effects formula:

$$\alpha = 1 + 2.367b^{0.867} \tag{2}$$

Given the substantial agreement between α values for disk and square capacitors, and considering that edge effects are quite small for the aspect ratios here (e.g., for b = .005, $\alpha = 1.0239$), we use Eq. (2) to approximate edge effects. Any error resulting from extension of this formula to our data should be much less than 1% and can be ignored.

B. Tilt Effects

Because actual capacitor plates cannot be made perfectly parallel, there is a tilt effect not reflected by Eq. (1). In our experiments, a plate separation of $d = 0$ signifies that the plates just begin to touch. This point was found by finely adjusting the plate separation distance until the resistance between the plates became zero as measured by a multi-meter. Accordingly, d represents the minimum distance between the plates, not the average distance, which is somewhat larger as a result of tilt. If Δd is the average deviation in the plate separation distance above d, then a correction can be given by,

$$\Delta C = \epsilon_0 A \Delta(1/d) = -\epsilon_0 A \frac{\Delta d}{d^2} = -C\frac{\Delta d}{d}. \tag{3}$$

This correction is valid to the first order and is best where $\Delta d/d$ is small. However, for very small d, even minor plate tilt can be non-negligible.

C. Body Capacitance

In addition to the capacitance due to the inner faces of the plates, the apparatus has a small "body capacitance" of roughly 3 pF which is found by measuring the capacitance with the plates far apart. This body capacitance reflects the residual capacitance due to, e.g., the apparatus frame, probes, and the thickness of the plates and does not vary significantly with plate separation distance.

D. Corrected Formula for Capacitance

Accounting for both edge effects and plate tilt as well as body capacitance, actual capacitance can be modeled by,

$$C = C_{body} + \alpha\left(C_0 - C_0\frac{\Delta d}{d}\right) = C_{body} + \alpha\left(\epsilon_0\frac{A}{d} - \epsilon_0\frac{A\Delta d}{d^2}\right). \tag{4}$$

This reflects that actual capacitance, like idealized capacitance, should vary linearly with plate overlap area. However, capacitance varies only roughly as $1/d$. At very small d, where tilt effect is exaggerated but edge effects are minimal, capacitance is smaller than idealized, while at larger d, where tilt effect is small but edge effects are bigger, capacitance can be larger than idealized.

IV. INVESTIGATIONS CONDUCTED WITH THE APPARATUS

Two experiments are performed to test the ideal capacitance equation obtained from Gauss's Law. Experiment I tests the relationship between capacitance and plate overlap area, while Experiment II tests the relationship between capacitance and plate separation distance. In both experiments, the space between the plates is filled only with air. Capacitance is measured using a meter with a sensitivity of +/- 1 pF. After correcting for edge effects, tilt effects, and body capacitance, measurements are found to consistently agree with Eq. (1) within 3%, providing strong empirical confirmation of Gauss's Law.

A. Experiment I: Variable Area Experiment

The purpose of the first experiment is to confirm that measured capacitance is proportional to plate overlap area, or $C \propto A$ as seen in Eq. (1) and Eq. (4). To minimize the tilt effects, plate separation is held constant at a sufficiently large value that $\Delta d/d \approx 0$. The lower plate is slid horizontally along a fixed axis to adjust the overlap area of the parallel plates, and capacitance data is gathered. Measured capacitance is plotted against

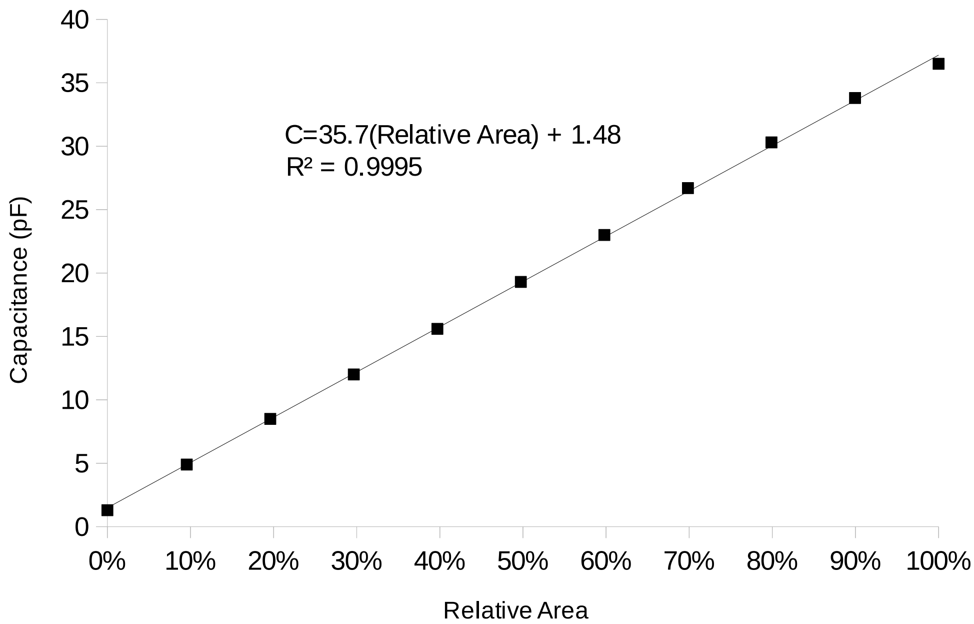

FIG. 2. Measured capacitance graphed as a function of the relative area.

relative area (A/A_{max}), and a regression is performed to test for linearity. The results, as depicted in Fig. 2, show a strong linear relationship between capacitance and plate overlap area (statistical $R^2 = .9995$).

B. Experiment II: Variable Distance Experiment

The purpose of the second experiment is to verify the relationship between capacitance and plate separation distance and, after correcting for effects, to test the validity of the ideal capacitance equation derived from Gauss's Law. Capacitance data is collected over a plate separation range of 0.2-1.6 mm using the following increments: 10 µm (from 0.2-0.4 mm), 20 µm (from 0.4-0.8 mm), and 40 µm (from 0.8-1.6 mm). The lower end of the range is selected to limit tilt effects and keep measurement errors due to the 10 µm precision limit of the micrometer head to less than 5%. The upper end of the range is chosen so that edge effects are relatively small and can be accurately modeled with Eq. (2). Three trials are conducted, and the capacitance values at particular plate separation distances are averaged.

As revealed by Fig. 3, before accounting for tilt effects, edge effects, and body capacitance, measured capacitance differed from ideal capacitance by as much as 15%. On the low-end

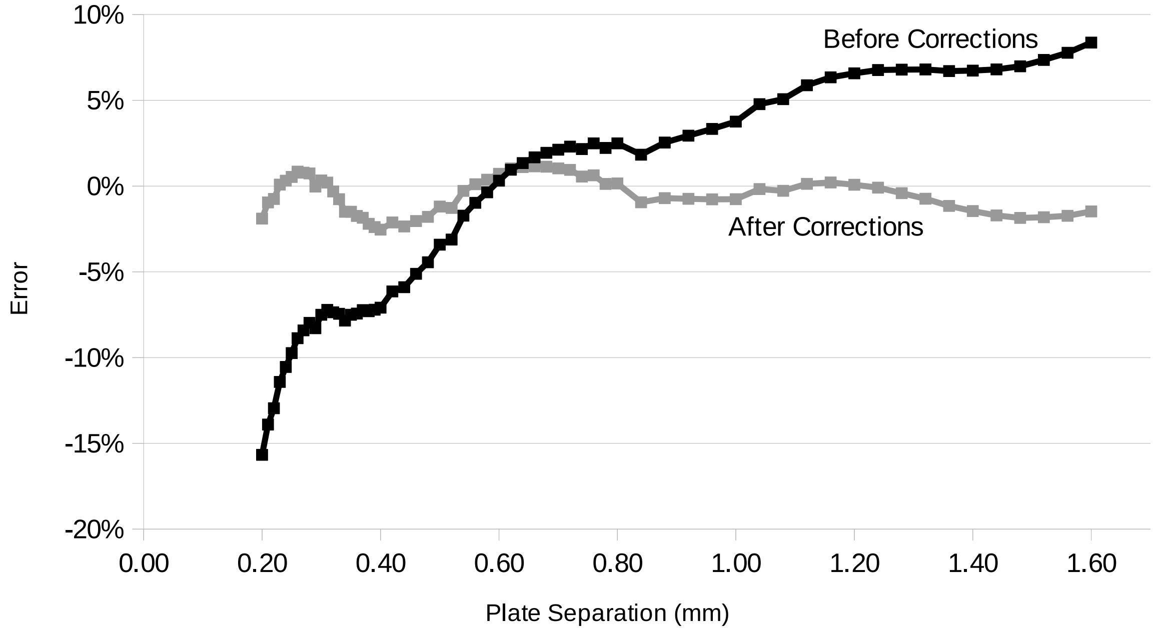

FIG. 3. Percent error as a function of plate separation before and after corrections are made.

of the plate separation range, where the aspect ratio is extremely small, deviations are due primarily to tilt, resulting in measured capacitances that are significantly lower than that given by Eq. (1). At the high-end of the plate separation range, where tilt effect is small but edge effects are increased, measured capacitances are higher than idealized. After correcting for these effects, as detailed below, capacitance values are found to closely correspond to ideal values with errors ranging from -3% to +1% and an average absolute error of approximately 1%.

1. *Error Analysis*

The first step in the correction process is to subtract the 3 pF body capacitance from the measured values. Next, edge effects are removed by dividing data points by the values of α obtained from Eq. (2). Let the partially corrected values be denoted by C'. A multiple regression is then performed of the form,

$$C' = (C - C_{body})/\alpha = \epsilon_0 \frac{A}{d} - \epsilon_0 \frac{A \Delta d}{d^2} = \beta_1 d^{-1} + \beta_2 d^{-2}. \quad (5)$$

This yields coeffecients (95% confidence interval) of $\beta_1 = (8.80 \pm 0.08) \times 10^{-14}\,\mathrm{F\,m}$ and $\beta_2 = (-2.70 \pm 0.22) \times 10^{-18}\,\mathrm{F\,m^2}$. The first coefficient, β_1, is in good agreement with the value of $\epsilon_0 A$ (or $8.85 \times 10^{-14}\,\mathrm{F\,m}$). Meanwhile, the average deviation in plate separation, Δd, is found from the ratio:

$$\Delta d = \beta_2/\beta_1 = 3.07 \times 10^{-5}\,\mathrm{m} = 30.7\,\mu\mathrm{m}. \tag{6}$$

Removing the term varying as d^{-2} (the tilt effect), we are left with corrected capacitances that can be compared to the ideal ones given by Eq. (1). These show a remarkably good fit to the ideal capacitance formula as shown in Fig. 3, with corrected capacitance values consistently within 3% of ideal. These corrected capacitances are also seen to vary inversely with plate separation distance, as predicted for an ideal capacitor (see Fig. 4).

Sources of error include the capacitance meter and the micrometer head. Measurement errors attributable to the capacitance meter are most significant at large plate separation distances but never exceed 2% (much less than 1% in the middle of our plate separation range). By contrast, errors in distance measurements using the micrometer head are most significant for small plate separation distances and can be as large as 5% but are typically much smaller (approximately 2% in the middle of our plate separation range). These account for most of the small discrepancy between the corrected and ideal capacitance values.

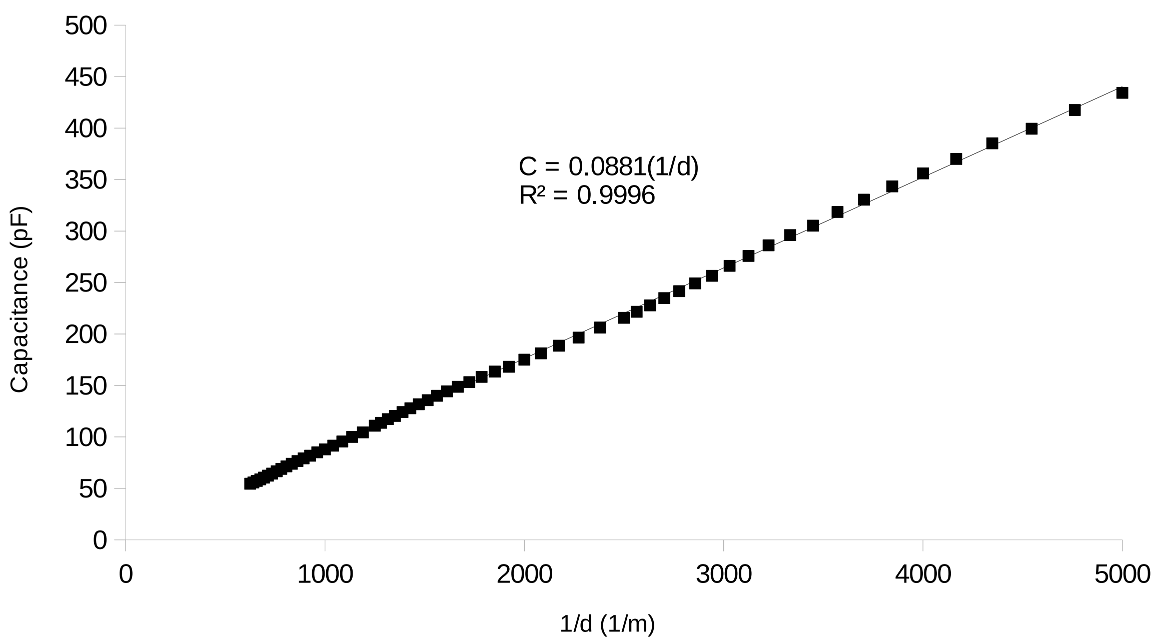

FIG. 4. Corrected capacitance graphed as a function of $1/d$.

V. CONCLUSION

We have designed a precision parallel plate capacitor apparatus allowing for fine variation of separation distance and plate overlap area for use in an undergraduate laboratory or for field work. Further, we have demonstrated two of the experiments that can be performed with the device. After correcting for tilt effects, edge effects, and body capacitance, our experiments yield results in strong agreement with the ideal capacitance formula, confirming Gauss's Law.

ACKNOWLEDGMENTS

We gratefully acknowledge the QuarkNet program and Southern Methodist University, whose support made this research possible, and eCube Instruments LLC, which produced the adjustable parallel plate capacitor apparatus designed by Jingbo Ye.

---

* beau_wells@allenisd.org

† yejb@physics.smu.edu

[1] Shiree Burt, Nathan Finney, and Jack Young, "Fringe Field of Parallel Plate Capacitor,"<http://santarosa.edu/~yataiiya/UNDER_GRAD_RESEARCH/Fringe%20Field%20of%20Parallel%20Plate%20Capacitor.pdf>

[2] K.P.P. Pillai, "Fringing field of finite parallel-plate capacitors," *Proc. IEE*, vol. 117, no. 6, pp.1201-1204, June 1970.

[3] Hitoshi Nishiyama and Mitsunobu Nakamura, "Form and Capacitance of Parallel-Plate Capacitors," *IEEE Trans. Compon. Packag. Manuf. Technol.*, vol. 17, No. 3, pp.477-484, Sep. 1994.

[4] Daniel Kinseth Reitan, "Accurate Determination of the Capacitance of Rectangular Parallel-Plate Capacitors," *J. Appl. Phys.*, vol. 30, pp.172-176, Feb. 1959.

[5] G.W. Parker, "What is the Capacitance of Parallel Plates?," *Comput. Phys.*, vol. 5, pp.534-540, Sep/Oct. 1991.

[6] G.W. Parker, "Electric field outside a parallel plate capacitor," *Am. J. Phys.*, vol. 70, no. 5, pp.502-507, May 2002.

[7] G.T. Carlson and B.L. Illman, "The circular disk parallel plate capacitor," *Am. J. Phys.*, vol. 62, No. 12, pp.1099-1105, Dec. 1994.

[8] Olivier Schneegans, Pascal Chrétien, Frédéric Houzé, and René, "Capacitance measurements on small parallel plate capacitors using nanoscale impedance microscopy," *Appl. Phys. Lett.*, vol. 90, 043116, Jan. 2007.