Calculating the Relative Permittivity Constants of Various Dielectric Materials Using a Parallel Plate Capacitor

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Abstract

Two experimental setups involving an adjustable and fixed parallel plate capacitor are used to measure the relative permittivities (ϵ_r) of paper and two oils. Adjustments for errors, such as edge effects, body capacitance and tilt effects, are performed for both experiments. The resulting relative permittivity value for paper falls within an accepted range, and the relative permittivities of the oils fall within 3% of verified values. The accuracy of these experiments after corrections allows for hands-on inquiries to be conducted in the classroom on an otherwise abstract concept in electrostatics.

I. INTRODUCTION

A common, albeit error-prone, electrostatics experiment is to study the relationship between the capacitance of a parallel plate capacitor and the dielectric. The equation describing the capacitance of an ideal capacitor with a dielectric is:

$$C = \frac{\epsilon_r \epsilon_0 A}{d} \tag{1}$$

where C is capacitance, A is plate overlap area, d is separation distance of the plates, and ϵ_r is the relative permittivity of the dielectric material. This relation lends itself to investigations on the effects of variable areas, plate separations, and the addition of dielectric materials within the capacitor. However, Eq. (1) applies only to ideal capacitors - devices comprised of two conducting plates that are infinitely large, infinitely thin, and perfectly parallel.

Measured capacitance values differ from the ideal because of multiple factors, as detailed by Wells, et al.[1] In addition, when using dielectrics, it is assumed that the material fills the entire gap between the parallel plates. Unfortunately, this ignores the effects of air between the plates and the dielectric and is not practical for any dielectric material whose thickness is on the order of the capacitor's tilt.

In this study, two experiments are performed. The first is to find the relative permittivity of a thin, solid dielectric material: paper. Calculating an accurate value presents a challenge for several reasons. The thickness of the paper, humidity of the environment, and variable composition of paper all have an effect on its relative permittivity.[2] To keep these conditions constant, only printer paper from the same package is used, and only measurements taken from the same day are compared. Paper's roughness also results in air gaps that must be incorporated into our calculations. The second experiment studies liquid dielectrics and a method to calculate their relative permittivities. An issue that must be considered is the capillary effect between the metal plates and the liquid dielectric. The calculated relative permittivities are then applied to find the depth of the liquids in a container. This method can be used to find the depth of liquids in opaque containers or in situations where the depths cannot be measured directly.

II. DEVICE DESCRIPTION

Experiment 1 utilizes a horizontally and vertically adjustable cm x 10 cm parallel plate capacitor with a 2.0 mm plate thickness. The top plate is fixed to an insulating frame, while the bottom plate is attached to a mobile optical stage. The overlap area of the plates can be adjusted from 0 cm^2 to 100 cm^2. The separation between the two plates can be set using an adjustable micrometer head with 0.01 mm precision from 0 mm to 10 mm. The accuracy of this device is discussed by Wells, et al.[1]

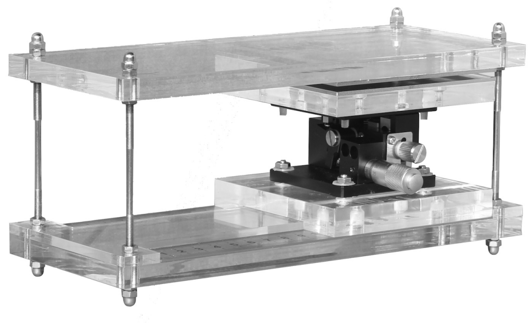

FIG. 1. The instrument used in Experiment 1.

The capacitor in Experiment 2 has two 10 cm x 10 cm metal plates of roughly a 1.0 mm fixed separation attached by four acrylic screws. An open acrylic container (12.5 cm x 2.5 cm x 12.5 cm) holds the capacitor plates and the liquid under study.

FIG. 2. The instrument used in Experiment 2.

The capacitance in each experiment is measured using a digital capacitance meter capable of measuring down to tenths of picofarads with a relative precision of 0.5%.

III. APPARATUS ERRORS

A. Edge Effects

Edge effects, or fringe effects, describe the accumulation of charge at the edge of the capacitor's plates, resulting in non-uniform charge densities. Due to these edge effects, the electric field between the plates is less than expected for an ideal capacitor and thus the capacitance is increased. Edge effects can be represented by a ratio of measured capacitance to ideal capacitance, $\alpha = \frac{C}{C_0}$.[1,3,4] A method for calculating this ratio is given by Nishiyama and Nakamura, where b is the aspect ratio of the plate separation to the side length.[3]

$$\alpha = 1 + 2.367b^{0.867} \tag{2}$$

Since the aspect ratios used in this experiment fall within the range of $b \leq 0.1$, this correction should produce little error.[1,8]

B. Tilt Effects

Distance separation varies across the plates since they cannot be perfectly parallel. Plate separation d is set at zero for the distance at which the capacitor's resistance reaches zero. However, this only signifies the minimum separation; the average distance is greater. The error due to tilt effect can be represented by the ratio $\frac{\Delta d}{d}$ by defining Δd as the average deviation in plate separation distance above d. The instrument manual gives Δd as 0.03mm.[5] This value is calculated to range between 0.03 to 0.07mm for our devices.

C. Body Capacitance

Though it is relatively small, extra charge from the apparatus' materials causes an increase in measured capacitance values. In Experiment 1, the body capacitance is calculated by removing the bottom stage from the apparatus to eliminate the possibility of an electric field that could accumulate at the edge of the plates when separated inside of the apparatus.

The body capacitance in Experiment 1 ranges from approximately 5 pF to 7 pF. In Experiment 2, the capacitance of the metal plates outside of the acrylic box is subtracted from the capacitance of the metal plates inside the acrylic box. The body capacitance measured ranges up to 1.8 pF.

D. Capillary Action

Liquids have the ability to flow against gravity due to strong intermolecular forces, adhesive properties, and cohesive properties. This effect is prominent when liquids are placed in narrow spaces and can be a significant source of error when dealing with liquid dielectrics.

E. Screws

In the fixed plate capacitor, the four acrylic screws that secure the metal plates together change the capacitance as a result of their dielectric properties and the area that they occupy, approximately 1% of the total area of the plates. In addition, the screws obstruct measurements of the capillary action between the plates.

IV. EXPERIMENT 1: RELATIVE PERMITTIVITY OF PAPER

This experiment's purpose is to measure the relative permittivity of paper using the adjustable parallel plate capacitor. Previously, Grove, Masters, and Miers use a self-designed capacitor of particle board and aluminum foil to calculate Teflon's relative permittivity. They choose Teflon rather than paper because of paper's varying ϵ_r value.[2] Our experimentation allows precise measurements of a thin dielectric to be obtained and systematic errors to be accounted for, which they acknowledge but do not consider.

The paper's thickness, as measured by a caliper, is 100 µm, which is in range of the tilt effect. It would be inaccurate to follow the conventional method of sandwiching the paper between the plates because of the paper's thinness.[6] With the tilt effect, there could still be a large air gap between the plates and the paper, which may cause errors in excess of 30%. If instead several sheets of paper are used to fill the separation between the plates, then the air gaps between the sheets of paper must still be accounted for in addition to the tilt effect.

To limit these effects, the plates are separated at distances from 0.5 mm to 4.0 mm (using 0.5 mm increments) and a varying number of sheets of paper is placed on the bottom plate. Up to half of the maximum number of papers that can fit into the plate separation is used to leave an air gap that minimizes the tilt effect error. Plate overlap is kept at a consistent 100 cm^2. As seen from Fig. 4 from Ref. 2, the capacitance through the air gaps and pieces of paper can be combined and treated as individual blocks of dielectric respectively. The measured capacitance (C_m) is thus defined as the capacitance of each block - capacitance through air (C_a) and through paper (C_p) - connected in series.

$$\frac{1}{C_m} = \frac{1}{C_a} + \frac{1}{C_p} \tag{3}$$

where C_m is measured capacitance, C_a is capacitance through air, and C_p is capacitance through paper. In terms of the capacitor's physical properties, this can be written as:

$$\frac{1}{C_m} = \frac{d_0 - nd_1}{\epsilon_0 A} + \frac{nd_1}{\epsilon_r \epsilon_0 A} \tag{4}$$

$$\frac{1}{C_m} = \frac{nd_1}{\epsilon_0 A}(\frac{1}{\epsilon_r} - 1) + \frac{d_0}{\epsilon_0 A} \tag{5}$$

where d_0 is the plate separation, d_1 is the thickness of one sheet of paper, n is the number of sheets of paper, and A is the plate overlap area. When $\frac{1}{C_m}$ is plotted versus n, the slope m of that graph can be represented as:

$$m = \frac{d_1}{\epsilon_0 A}(\frac{1}{\epsilon_r} - 1) \tag{6}$$

Eq. (6) can then be manipulated to solve for ϵ_r.

$$\epsilon_r = \frac{d_1}{\epsilon_0 A m + d_1} \tag{7}$$

This method yields an ϵ_r of 2.451, considerably smaller than the given 3.7 value.[7] Therefore, multiple combinations of error corrections are applied to the measured capacitance in order to find a more accurate relative permittivity value.

A. Error Corrections

In order to improve upon the initial value obtained from Experiment 1, edge effects, body capacitance (C_{body}), and tilt are accounted for in the measured capacitance value.

First, body capacitance is subtracted from the measured capacitance. These values are then divided by the α values obtained from Eq. (2)[1,3] to remove edge effects. Tilt effects are then placed into consideration using a first-order linear correction from Wells, et al.[1] The final corrected formula is

$$C' = \frac{C_m - C_{body}}{\alpha(1 - \frac{\Delta d}{d})} \tag{8}$$

where C_m is measured capacitance, and C' is corrected measured capacitance.

The reciprocal of C' with respect to n, is graphed and fitted with a linear trend-line. Using Eq. (6) yields the relative permittivity of paper.

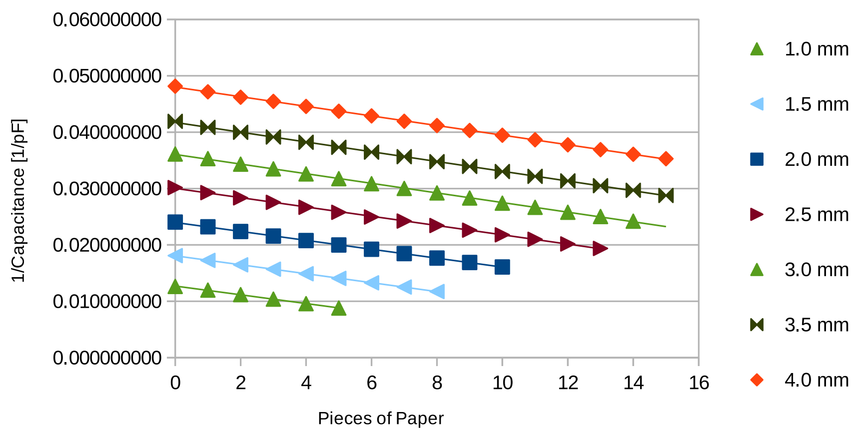

FIG. 3. The reciprocal of corrected measured capacitance as a function of the number of sheets of paper added. Plate separation distance for each series is indicated in the legend.

Table I indicates that each correction significantly reduces the error in comparison to the uncorrected value for paper's ϵ_r. When the edge effects are corrected, the error falls below 10%, demonstrating that edge effects are not negligible. The most accurate ϵ_r value is obtained from accounting for all of the errors. This gives evidence that Ref. 1 and above calculated corrections provide valid ϵ_r values when using our parallel plate capacitors.

B. Conclusions

After corrections, the relative permittivity of paper is found to be about 3.780. It is found that due to manufacturing differences, the humidity of the air, varying compositions of paper, etc., the relative permittivity can actually vary between 1.7 and 4.0.[2,7] Therefore,

Combination of Error Adjustment Corrections	Measured Relative Permittivity (Averaged Over Multiple Trials)	Percentage Error from 3.7
No Errors Corrected	2.451	33.76%
Only Body Capacitance Corrected	3.114	15.84%
Edge Effects and Body Capacitance Corrected	3.990	7.838%
Tilt Effect and Body Capacitance Corrected	3.027	18.20%
All Errors Corrected	3.780	2.162%

TABLE I. Resultant calculated relative permittivity values according to measurements after varying combinations of error adjustments and their percent differences from a reference value.

3.780 is a valid value for the relative permittivity of paper.

V. EXPERIMENT 2: RELATIVE PERMITTIVITY OF OIL

A.

The purpose of this experiment is to determine the relative permittivities of soybean and canola oil. A fixed capacitor is placed into an empty acrylic container and its capacitance is measured. The container is then filled with oil, and the capacitance is measured again. The idealized equations for the capacitor completely filled with air and oil are given below, respectively:

$$C_{air} = \epsilon_0 \frac{A}{d} + C_{body} \tag{9}$$

$$C_{oil} = \epsilon_r \epsilon_0 \frac{A}{d} + C_{body} \tag{10}$$

The ratio of these equations solves for the dielectric constant ϵ_r.

$$\frac{C_{oil} - C_{body}}{C_{air} - C_{body}} = \epsilon_r \tag{11}$$

After several trials and correcting for body capacitance, the relative permittivity of canola oil and soybean oil are measured to be 3.16 and 3.22, respectively. According to Fig. 1 from Ref. 9, the relative permittivity of these oils at room temperature (25 °C) are 3.08 and 3.13, which gives an error of 2.60% and 2.88%.

B.

The dielectric constant obtained from Part A is used to calculate the oil depth based on the measured capacitance. The calculated depths are then compared to the measured values. In the experiment, the parallel plate capacitor can be considered as two capacitors in parallel, one with an oil medium and the other with an air medium. The measured capacitance is then a sum of the two capacitances:

$$C_m = C_{air} + C_{oil} \tag{12}$$

$$C_m = \frac{\epsilon_0 l(l-h)}{d} + \frac{\epsilon_0 \epsilon_r l h}{d} \tag{13}$$

where l is the side length, h is the oil depth, and d is the plate separation. Eq. (13) can then be manipulated to solve for oil depth h:

$$h = \frac{1}{\epsilon_0 - 1}\left(\frac{C_m d}{\epsilon_0 l} - l\right) \tag{14}$$

The percent difference between the calculated and measured depths is graphed in Figure 4a. As shown in the figure, there is a significant discrepancy between the measured and calculated oil depths. The trials are conducted again after capillary action is observed (as pictured by Figure 5), which differs from the measured oil depths by up to 5.0 mm. Instead, comparing the calculated results to the measured capillary depths results in a large decrease in error as seen in Figure 4b. However, this difference is still quite large and leads to questions of other possible sources of error, such as the cleanliness of the plates and actual plate separation. Plate cleanliness is tested by disassembling one of the capacitors, cleaning the residual oil from between the plates, and allowing it to dry overnight. The capacitor is reassembled and the measured capacitance is compared to values from previous measurements. The residual oil is found to have a negligible effect. The plate separation is then measured and found to be slightly less than 1 mm for each capacitor. Unfortunately, the exact plate separation varies on each side of the capacitor by a slight amount. The average separation is instead calculated using the ideal capacitance formula with corrections for body capacitance and edge effects. Using this corrected plate separation in Eq. (14) produces the results seen in Figure 4c.

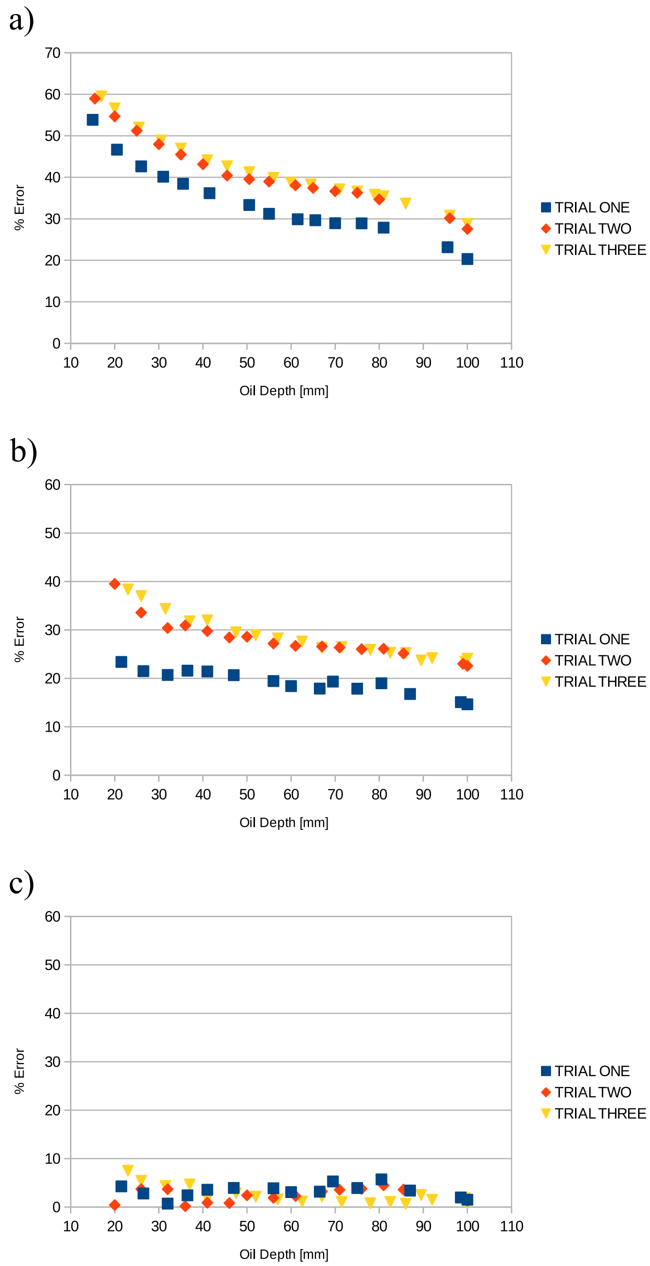

FIG. 4. Percent difference between calculated oil depth from capacitance values and measured oil depth as a function of measured oil depth. Each graph includes extra error corrections compared to its immediate predecessor, as described in section V B.

C. Experimental Conclusion

After accounting for all of the applicable issues, the mean error of the measured and calculated oil depths, where ϵ_r is equal to the accepted value for each oil tested, is 2.83%.

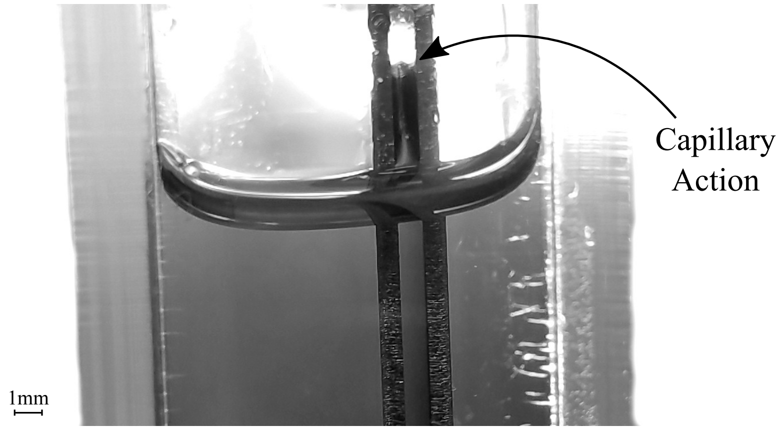

FIG. 5. Side image of experimental setup, with capillary action indicated.

We expect that the remaining error is attributed to the screws in the capacitor plates.

VI. APPLICATIONS/CONCLUSIONS

The results from both experiments produce accurate values for the relative permittivities measured for paper, canola oil, and soybean oil. After corrections, the calculated ϵ_r are within 3% of their respective values. This proves that the apparatus and methods utilized to correct the data can work with a variety of materials. We have demonstrated that Experiment 1's methodology accurately produces results for thin dielectric constants. Calculating the oil depth in Experiment 2 presents a novel way of measuring depth of other liquids, especially those in opaque containers or opaque liquids themselves. A continuation of this research might involve calculating the relative permittivity of the acrylic screws and finding a reliable method to approximate capillary action. This lab provides a hands-on demonstration for dielectrics in a classroom while also applying an ideal formula in a nonideal environment.

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