General Physics - E&M (PHY 1308) Lecture Notes

Lecture 003: Electric Field and Simple Distributions of Charge (Wolfson 20.3-20.4)

SteveSekula, 26 August 2010 (created 19 August 2010)

no tags

Main Goals of this Lecture

- Introduce the principle of "superposition"
- Review the field concept and extend it to the electric force and electric charge
    - Discuss the field of a point charge
- Discuss the fields due to distributions of charge
    - Discuss the electric dipole, one of the most important "simple distributions"

Relevant Physics Simulators:

- "Electric Field Hockey": http://phet.colorado.edu/en/simulation/electric-hockey Can you use electric charge, correctly positioned, to steer the ball into the goal?
- "Electric Field of Dreams": http://phet.colorado.edu/en/simulation/efield Explore the effect of various electric charges not just on each other, but on the electric field at each point in space

Problem Solving: Coulomb's Law

To attack a problem involving Coulomb's Law, you need to keep a few definitions in mind:

- $\vec{F}_{12}$ is the *force that charge 1 exerts on charge 2*
- q_1 is the charge of the source charge (and is a signed quantity) and q_2

is the charge of the target charge (the one on which you are trying to determine the force)

- the unit vector $\hat{r}$ always points from the *source charge* to the *target charge*
- double-check any results using what you know about charges:
  - like charges REPEL
  - unlike charges ATTRACT

Let's setup a problem:

QUESTIONS: A 1.0-μC charge is at $x = 1.0\mathrm{cm}$, and a -1.5-μC charge is at $x = 3.0\mathrm{cm}$. What force does the positive charge exert on the negative one? How would the net force change if the distance between the charges tripled?

INTERPRET: We identify the $q_2 = -1.5$-μC as the one on which we want to find the force, and thus the $q_1 = 1.0\mu\mathrm{C}$ charge is the *source charge*.

DEVELOP: We're given coordinates, so let's draw a picture and label things. The nice part about this is that the charges lie on the same axis (the x-axis, in this case). With the source charge (q_1) to the left of q_2, the unit vector in the direction from q_1 to q_2 is $\hat{i}$.

EVALUATE: Now we use Coulomb's Law to evaluate the force:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r} = \frac{(9.0\times10^9\mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2)(1.0\times10^{-6}\mathrm{C})(-1.5\times10^{-6}\mathrm{C})}{(0.020m)^2}\hat{i} = -34\hat{i}\mathrm{N}.$$

This force is at a separation of $2\mathrm{cm}$. If that distance tripled to $6\mathrm{cm}$, then the force would scale by

$$F'_{12}/F_{12} = r_{12}^2/(r'_{12})^2 = (2\mathrm{cm})^2/(6\mathrm{cm})^2 = 1/9$$

bringing the force at $6\mathrm{cm}$ separation to $\vec{F}'_{12} = -3.8\hat{i}\mathrm{N}$.

Point Charges and the Principle of Superposition

When dealing with more than one pair of charges, you need a strategy for computing the force on a charge, Q, given a number of other charges q_i (where i runs from 1 to N and labels each of the remaining charges). Because force is a vector, to find the total force on Q you add the forces (vectors) exerted on Q by the charges q_i. The force that q_1 exerts on Q is unaffected by the force q_2 exerts on Q - this allows us to superpose the individual forces to find the total force. This is not obvious, but its reality has been upheld by experiments and observations of nature. Nature didn't have to be this simple, but it is.

Why do you need this? Coulomb's Law applies to **point charges** - charged objects whose size is negligible. However, the real world is populated by **charge distributions** - a collection of many charges spread out over space. For instance:

- molecules are an example of distributions of charges - protons and electrons - and those distributions matter when you are thinking about how different molecules interact with one another (and, since they are similarly sized, you cannot neglect their dimensions).
- your heart contains a charge distribution, which accumulates during systole (contraction of the heart) and causes heart muscle tissue to contract and pump blood

Therefore, we are often confronted with situations where we need to deal with a distribution of charge.

Review of the Field Concept

Forces like gravity bothered scientists in the 1600s because you had to invoke "spooky action at a distance" to explain how, for instance, the earth kept the moon in orbit. The idea of a field relieves the mind of the concern about a mysterious and unseen contact between two objects; instead, it introduces the idea that, for instance, the earth creates a gravitational field and the moon responds to that field.

In gravitation, we talk about the acceleration due to gravity. That can be written:

$$\vec{g} = \vec{F}/m.$$

The gravitational acceleration can then be thought of as the force per unit mass that an object in Earth's gravitational field would experience. $\vec{g}$ becomes the *gravitational field*, and it is defined as the force per unit mass at any point in space around the mass.

Electric Field

We define the electric field similar to the way that you have learned about the gravitational field. It was Michael Faraday (1792-1867) [http://en.wikipedia.org/wiki/Michael_Faraday] who introduced the idea of an electric force field. Again, he did so to explain the "spooky action at a distance" that objects appeared to experience in the presence of electric charge.

Demonstration: the Van de Graaff Generator

If you've never felt an electric field before, after this you'll believe they exist.

Describing the electric field

The electric field is given by:

$$\vec{E} = \vec{F}/q,$$

the *force per unit charge* experienced by a charged object at any point in space. The electric field exists everywhere in space, and we represent that field by a series of vectors showing the force experienced by a charge q at the corresponding points in space.

Explore the electric field concept through visualization:

- http://phet.colorado.edu/en/simulation/efield

The idea of a field can be quite abstract, at first, but it's a useful idea that pervades physics (in fact, "Quantum Field Theory" is the underlying mathematical description that we have of nature). In the laboratory, you can map out the electric force field by measuring the electric force over a large number of points around a point charge, or a series of charges (e.g. between two sheets of charge).

We have to be a little careful with the field concept. The q in the field equation above is assumed to be small relative to the charge, or distribution of charge, that we are probing. This is so that the field of the charge itself can be neglected. This "test charge" idea is useful for visualization, but be careful with it.

Connecting Coulomb's Law of Force to the Electric Field concept

We have a concept of the electric field. Let's connect it to Coloumb's Law and determine the mathematical formula for the **electric field of a single point charge**:

$$\vec{E} = \vec{F}/q = \left(\frac{kQq}{r^2}\hat{r}\right)\frac{1}{q}$$

The units of electric field are N/C (Newtons per Coulomb).

- Fields of hundreds to thousands of N/C are commonplace
- Fields of 3 MN/C (3×10^6 N/C) will tear the electrons off of air molecules

The above equation for the electric field is so closely related to Coulomb's Law that it is often referred to simply as "Coulomb's Law." Since $\hat{r}$ always points away from Q, the electric field extends OUTWARD from positive charge and INWARD to negative charge.

The Electric Fields of Charge Distributions

More commonplace in nature than a single charge are *distributions of charge*. Just as with the aggregate force from a set of charges, the combined electric field from a distribution of charge is obtained by the vector sum of their individual electric fields:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... = \sum_{i=1}^{N} \vec{E}_i = \sum_{i=1}^{N} \frac{kq_i}{r_i^2}\hat{r}_i.$$

The Electric Dipole

An electric dipole is simply an object consisting of **two point charges of equal magnitude by opposite sign**. The ends are joined in some rigid mechanical way (e.g. through a physical expanse of material, or a chemical bond).

Examples of electric dipoles:

- water molecules
- Radio and TV antennas
- The heart (during systole, or the contraction phase).
    - an electrocardiogram is a measure of the strength of the heart dipole that forms when it contracts (which happens as cells move ions around to create a charge imbalance)

An electric dipole has more structure than a single, lonely charge:

- Its net charge is ZERO
- It still possesses an electric field, owing to the *slight separation* of the two charges. Far enough away, the dipole separation becomes so small that it is negligible, and it should appear electrically neutral.
- Have a look at Example 20.5 in the book

A dipole is more complex. We wish to describe its aggregate behavior (the behavior of the whole system). As such, we have developed the idea of a **dipole moment** to describe the whole system:

- The **dipole moment** points from the negative to the positive charge, and has a magnitude $p = qd$, where q is the magnitude of either of the two charges and d is the separation between the two charges.
- A typical dipole problem will involve finding the field far from the dipole, or finding the force on the dipole exerted by a charge that is far from the dipole. In the language of math, we express this as $r >> d$, where r is the distance from the mid-point of the dipole to the external location (another charge, or a point P) and d is the separation of the charges in the dipole.