General Physics - E&M (PHY 1308) Lecture Notes

Lecture 005: Electric Field Lines and Electric Flux

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no tags

Goals of Today's Lecture

- Discuss a representation of charge and its sign and magnitude via the use of graphical representations of the electric field
- Discuss the meaning of "flux" and its mathematical definition

Demonstration: Boston Museum of Science Lightning Show

Why is the man in this cage not killed instantly by this machine (think about how much the Van de Graaff hurt when it shocked you and compare to this behemoth)

We'll build a framework to answer this question, and in doing so we will begin to understand how to protect sensitive equipment from ambient electric fields (e.g. computers, etc.) and whole houses from ferocious lightning strikes.

As with many things in science, houses were protected from lightning (e.g. Franklin's lightning rod) before anybody actually understood why it worked, but the synthesis of invention, experimentation, and theoretical physics on electricity laid the groundwork for the modern world.

Electric Field Lines

To build this framework, we'll begin where most good things begin: with a nice picture.

We can visualize the electric field by drawing *electric field lines* emanating from (to) a positive (negative) charge. We've drawn these already, but now we're going to cement a standard for representing charge using electric fields.

The simplest way to draw fields is to employ Coulomb's Law. Pick a point in space (call that point P) around a charged object. Compute the electric field vector at that point. Now take a small step $\delta\hat{r}$ in the direction of the vector, to a new point P'. Now recompute the electric field vector at that point. Repeat. This traces out a part of the field. Pick another random starting point, Q. Repeat.

We can ask computers to do this quite easily, and in doing so generate a representation of the electric field (see slides).

What do we know about electric field lines so far?

- We know that they BEGIN on positive charges
- We know that the END on negative charges

What else can we learn about the representation of electric fields from Coulomb's Law?

- Electric fields are stronger the closer we are to the charge (e.g. think of a point charge). When we draw electric field lines, we see that they are closer together near the charge than further away
  - the density of electric field lines, given some fixed choice for the number of lines we draw, is representative of the strength of the field. More dense = stronger, less dense = weaker. Our image has mathematical and physical meaning.
  - consider the dipole field
- We know that stronger fields result from stronger charges. Let's associate that effect with the *number* of electric field lines we draw for a charge.
  - for a smallest charge q in a situation, associate that with 8 field lines
  - Then $2q$ is represented by 16, etc.

With this graphical convention in mind, we now have a tool for evaluating or representing physical situations. We can make statements about the relative strength of the electric field, and thus the electric force, at some point in space. We can represent different sized and sign charges. Etc.

Electric Flux and Field

Great. So now we have a graphical tool that we can use to represent the mathematical statements we've been developing - Coulomb's Law, for instance. Let's put this graphical technique to use and see what we can learn about field and charge.

Board Exercise:

Closed surfaces around charges: these are surfaces that don't allow entry unless you break the surface.

- Draw the field of a point charge, $+q$ (8 lines radiating out from the charge)
- Enclose that charge in a simple spherical surface (represented in the 2-D board by a circle

How many electric field lines leave the surface?

- Now enclose the charge in a larger surface, this one a goofy square or something like that. Ask the same discussion question again.
- Draw a really weird surface, this time with pinch points that allow field to leave the surface, enter it again, and then leave it. Ask the same question.
- Draw an additional surface that DOESN'T enclose a charge, and repeat the question.

Repeat this exercise with a charge $+2q$

What have we learned from this exercise? No matter how we enclose the charge with a closed surface, **the number of field lines emerging from a closed surface is proportional to the net charge enclosed by the surface**.

This statement is very general, and doesn't matter at all what surface looks like (so pick an easy one, if you can!). The presence of charges OUTSIDE the surface doesn't affect our conclusions about the charge enclosed in the surface.

Electric Flux

We can then turn this graphical tool back into a mathematical statement. First of all, we can describe the electric field lines emerging through a surface quite generally, by introducing the concept of "flux".

Imagine a square surface of area A through which a field passes. Let the field be depicted by four lines, perpendicular to the surface. We need a mathematical way of describing the orientation of this surface to the field. To do this, we can define a vector that emerges perpendicular to the surface, and call that the *normal vector* $\vec{A}$. This vector described the orientation of the surface with respect to the field. If we tilt the surface away from perpendicular to the field, the the normal then begins to form an angle with respect to $\vec{E}$.

When the plan is parallel to the field, no more electric field lines pass through the surface. That is, zero field lines pass through the surface. When the plane is perfectly perpendicular to the field, then the number of lines passing through the surface is maximal. Anywhere inbetween is a fraction of that maximum. So the angle of tilt is related to the amount of field flux passing through A.

Let's jot down a few of these observations, and a few more:

- Start with the surface of area A perpendicular to the field
  - if you double the electric field strength (doubling the field lines), you double the amount of electric field passing through the surface - this is the doubling of the flux, the "flow" of the electric field lines through the surface
  - if instead you cut the area of the surface in half, this reduces also by a factor of 2 the number of lines passing through the surface.
  - our first conclusiuon: the size of the flux is related to the magnitude of the electric field and the area of the surface
- Consider orienting the surface (characterized by the normal to the surface) relative to the field

  - the amount of flux is proportional to $\cos\theta$. When θ is zero, $\cos\theta = 1$ and the flux is maximal. when $\theta = \pi/2$, then $\cos\theta = 0$ and the flux is minimal (zero) through the area.

Thus the flux can be mathematically described as

$$\Phi = EA\cos\theta.$$

This may look familiar. This is just the magnitude of the dot product between the field and the normal to the area:

$$\Phi = \vec{E} \cdot \vec{A}.$$

- Flux and field are distinct things. The field is a vector defined at each point in space, but the flux is a number, a scalar, a global property that depends on how the gfield behaves at a surface rather than at a single point. It quantifies the number of field lines passing through a surface.

Open vs. Closed Surfaces

The surfaces in the previous discussion were open - that is, you could get "inside" them without penetrating the surface. In this case, the direction in which $\vec{A}$ points is ambiguous, and you have to make a choice and state it.

When the surface is CLOSED, however, the convention for $\vec{A}$ is unambiguous: **for a closed surface, $\vec{\mathbf{A}}$ points OUT of the surface**.

Non-Flat Surfaces

What about when the surface is not simple: e.g. is very curved? Then, instead of thinking about the whole surface we can think about little patches of the surface, $d\vec{A}$, each characterized by their own normal and each the same size. If the patches are small enough, then to a good approximation each patch is very flat and then we simply have to integrate over all patches to get the total flux:

$$\Phi = \int_{surface} \vec{E} \cdot d\vec{A}.$$

The limits range over the entire surface, picking up contributions from all pieces. Although the integral could, in principle, be very difficult to evaluate, we'll find it to be most useful in cases where the integral turns out to be more straight-forward (the book uses the word "trivial" - I HATE that word. It implies that if you fail to do it, you're somehow a lesser person. Math is a struggle for each in their own way, and not at all for some. It's best to instead realize that with time and practice, you can tackle these things.)

Gauss's Law

We've arrived at the moment when we can derive and then discuss one of the foundational, fundamental equations in physics. This equation is part of a set of a few equations - laws of nature - which formed the basis of the revolutions that led to quantum physics and relativity. But to get there, we have to derive, use, and try to understand this equation. It is called "Gauss's Law".

If there is time, start with the notion that the flux is related to the enclosed charge. From there, assume a point charge enclosed in a spherical surface. The integration over the surface yields just $\Phi = E(4\pi r^2)$ (since the normals all make zero angle with the field, $\cos\theta = 1$ for all $d\vec{A}$). Insert Coulomb's Law for the electric field, and find that $\Phi = 4\pi k q_{\mathrm{enclosed}}$. Define $\epsilon_0 \equiv 1/4\pi k$ and state that we'll use this form of k from now on. It has units $\epsilon_0 = 8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2$.