General Physics - E&M (PHY 1308) Lecture Notes

Lecture 008: Conductors, Electric Work and Potential

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no tags

Goals of this Lecture

- Discuss conductors, in lieu of Gauss's Law and real experiments
- Begin discussing potential and work.

Gauss's Law and Conductors

We have one last piece to discuss, regarding Gauss's Law and conductors. We need to define a conductor:

- A conductor is any material that permits the unrestrained and free flow of electric charge, either positive or negative. In a conductor, charges move with little or no resistance and so behave more like a collisionless gas free to move as fast as it can through the material. In fact, it's actually useful to think of metals, which are typically good conductors of electricity, not as a solid but as an ideal gas of electrons that are free to move under the application of a force. That simplification goes a LONG way to explaining the properties of metal - why they remove heat efficiently, why they bend and mold so easily without breaking (malleability), why they conduct electricity so well, etc.

So what happens inside a conductor when it's exposed to electric fields?

(See slides)

At first, the charges in the conductor respond to the external electric field. But in doing so, they eventually reach an overall configuration which balances the external electric field with an internal, but oppositely directed one. At this point, the net force on the charges is zero and they stop moving, apart from random thermal motion. This is called *electrostatic equilibrium*, and in this state the *net electric field inside the conductor is zero*.

This result is independent of the size and shape of the conductor, and could not be otherwise. If there was any internal electric field, there couldn't be equilibrium because the net force on a given charge would be non-zero.

This is a macroscopic view - a "bulk property" of the conductor. Of course, at the microscopic level there are still strong electric fields around individual electrons and ions in the conductor. But, the average field, taken over the entire conductor, is zero.

What about **charged conductors**? Consider a conductor that is in a region free from electric fields, and is thus neutral and in equilibrium. Conductors are neutral because they contain atoms with equal amounts of positive and negative charge. But, imagine introducing a *net electric charge* onto the conductor. For instance, consider that metal cage from the Boston Museum of Science demo, and the charge delivered to the metal cage by the lightning bolt from the Van de Graaff generator. What happens to that charge?

With an injection of electrons into a conductor, there is now a net NEGATIVE CHARGE present in the conductor. As a result, there is a net repulsive force inside the conductor, and the negative charges push each other apart. The furthest they can get away from each is to go to the surface of the conductor, at which point they continue to push each other apart on the surface until finally they can move no further and achieve equilibrium.

In an equilibrium state, there is no net electric field inside the conductor. Applying Gauss's Law to the problem at hand immediately demonstrates it. Imagining a gaussian surface drawn just inside the surface of the conductor, now in electrostatic equilibrium, there must be zero net electric field. Thus the enclosed charge must also be zero. It cannot be any other way, according to Gauss's Law. If there is a net charge on the conductor, it must lie outside the gaussian surface and so we conclude that it resides exactly on the surface of the conductor.

That's why the guy in the cage isn't killed. The charge from the lightning strike hits the conductor and immediately starts to jostle itself all across the outer surface of the cage, attempting to achieve maximal separation from itself. It quickly reaches electrostatic equilibrium (since the electrons are traveling at nearly the speed of light - $3.0 \times 10^8 \mathrm{m/s}$ - it takes no time at all to distribute themselves, even if many Coulombs are involved, over a cage of surface area that's just a few meters-squared. The person, their finger touching the INSIDE of the conductive cage, is free from the harmful effects of all of those charges trying to rush down his finger across his heart.

This business about all of the charge on a conductor moving to the outside of the conductor can be tested experimentally, as described in the book. Such tests have verified the inverse square law - that the power involved is, in fact, 2 - to some 16 decimal places.

Discuss lightning safety and electrical shielding of cables, etc. from stray electric fields.

Electric Potential Difference

Just like gravity, the electric force is *conservative* - that is, when you do work to move a charge against the field, you are storing potential energy. In gravity:

- We define a location (a height) as the "zero potential location." At this location, we simply define the potential energy to be zero
- If you then exert a force to lift an object from that point up higher in the gravitational field, to do this you must do work. In physics, work is energy and it is defined as the forced exerted over the distance it was exerted,

  $$W = \int \vec{F} \cdot d\vec{r} \rightarrow W = \int F\, dy \text{ (in one dimension).}$$

  If the force is constant over that distance (let's call it the height, h, to which the object is lifted), then the work is

  $$W = \int_0^h F\, dy = F \int_0^h dy = Fh = mgh.$$

In general, we discuss the *change in potential energy* in moving an object from point A to point B via a *force* as:

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}.$$

When the force doesn't vary over the path, we can more easily calculate the work:

$$W_{AB} = \vec{F} \cdot \Delta\vec{r}.$$

Let's focus on the electric field. Consider moving a positive charge q between points A and B (a distance Δr apart) in the presence of a uniform electric field $\vec{E} = -E\hat{\imath}$. Since the field is uniform, a constant electric force $\vec{F} = q\vec{E}$ acts on the charge, so:

$$\Delta U_{AB} = -W_{AB} = -q\vec{E} \cdot \Delta\vec{r} = -qE\Delta r \cos\pi = qE\,\Delta r$$

Pushing a positive charge AGAINST the flow of an electric field is like pushing a car uphill: potential energy INCREASES in both cases (it's larger at B than at A). Let go of the charge, and the electric force from the uniform field pushes it back to the left - just like letting the car go and watching it roll down the hill. The car is an analogy, one familiar in everyday life, that might be helpful when thinking about the electric force and potential energy.

Since ΔU is proportional to electric charge, it's useful to describe the potential energy change per unit charge between two points A and B:

$$\Delta V_{AB} \equiv \Delta U_{AB}/q = -\int_A^B \vec{E} \cdot d\vec{r}.$$

This is the **electric potential difference**, and its units are Joules/Coulomb, which is equal to the Volt. The Δ and subscripts indicate that we are EXPLICITLY TALKING ABOUT CHANGES IN POTENTIAL FROM ONE POINT TO ANOTHER.

- Strategy: in problems involving solving for changes in potential energy or in the electric potential difference, choose a point that you define as ZERO POTENTIAL ENERGY or ZERO POTENTIAL and work relative to that point to compute the difference.

In the special case of a uniform electric field, the above reduces to a simpler form:

$$\Delta V_{AB} = -\vec{E} \cdot \Delta\vec{r}.$$

where Δr is a vector from A to B.

- Potential difference can be either positive or negative, depending on whether the path goes against or with the field.
- Potential difference is a property *of two points*, and is INDEPENDENT of the path taken to get from A to B.