General Physics - E&M (PHY 1308) Lecture Notes

Lecture 009: Electric Potential, Work, and Energy

SteveSekula, 12 September 2010 (created 12 September 2010)

no tags

Goals of this Lecture

- Review the definition of work and potential
- Generalize to paths in non-uniform fields
- Discuss the potential in the "Boston Science Museum Lightning Show"

Electric Potential Difference

Just like gravity, the electric force is *conservative* - that is, when you do work to move a charge against the field, you are storing potential energy. In gravity:

- We define a location (a height) as the "zero potential location." At this location, we simply define the potential energy to be zero
- If you then exert a force to lift an object from that point up higher in the gravitational field, to do this you must do work. In physics, work is energy and it is defined as the forced exerted over the distance it was exerted,

  $$W = \int \vec{F} \cdot d\vec{r} \rightarrow W = \int F\, dy \text{ (in one dimension)}.$$

  If the force is constant over that distance (let's call it the height, h, to which the object is lifted), then the work is

  $$W = \int_0^h F\, dy = F \int_0^h dy = Fh = mgh.$$

In general, we discuss the *change in potential energy* in moving an object from point A to point B via a *force* as:

$$\Delta U_{AB} = U_B - U_A = -W_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}.$$

When the force doesn't vary over the path, we can more easily calculate the work:

$$W_{AB} = \vec{F} \cdot \Delta\vec{r}.$$

Let's focus on the electric field. Consider moving a positive charge q between points A and B (a distance Δr apart) in the presence of a uniform electric field $\vec{E} = -E\hat{i}$. Since the field is uniform, a constant electric force $\vec{F} = q\vec{E}$ acts on the charge, so:

$$\Delta U_{AB} = -W_{AB} = -q\vec{E} \cdot \Delta\vec{r} = -qE\Delta r \cos\pi = qE\,\Delta r$$

Pushing a positive charge AGAINST the flow of an electric field is like pushing a car uphill: potential energy INCREASES in both cases (it's larger at B than at A). Let go of the charge, and the electric force from the uniform field pushes it back to the left - just like letting the car go and watching it roll down the hill. The car is an analogy, one familiar in everyday life, that might be helpful when thinking about the electric force and potential energy.

Since ΔU is proportional to electric charge, it's useful to describe the potential energy change per unit charge between two points A and B:

$$\Delta V_{AB} \equiv \Delta U_{AB}/q = -\int_A^B \vec{E} \cdot d\vec{r}.$$

This is the **electric potential difference**, and its units are Joules/Coulomb, which is equal to the Volt. The Δ and subscripts indicate that we are EXPLICITLY TALKING ABOUT CHANGES IN POTENTIAL FROM ONE POINT TO ANOTHER.

- Strategy: in problems involving solving for changes in potential energy or in the electric potential difference, choose a point that you define as ZERO POTENTIAL ENERGY or ZERO POTENTIAL and work relative to that point to compute the difference.

In the special case of a uniform electric field, the above reduces to a simpler form:

$$\Delta V_{AB} = -\vec{E} \cdot \Delta\vec{r}.$$

where Δr is a vector from A to B.

- Potential difference can be either positive or negative, depending on whether the path goes against or with the field.
- Potential difference is a property *of two points*, and is INDEPENDENT of the path taken to get from A to B.

Units of Potential Difference

The units of electric potential difference are, based on the above, the $\mathrm{N} \cdot m/\mathrm{C} = \mathrm{J/C}$. This unit is conveniently called "the Volt".

What is a "Volt"? It's the work required to move a Coulomb from point A to point B. For instance, a 12V car battery is capable of moving 1C of charge across its terminals (REGARDLESS of the path the charge has to take - e.g. through the wiring in the car) using 12J of work:

$$12V = 12\mathrm{J/C}$$

That's the beauty of Voltage: it doesn't matter what path the charge has to take to go from A to B, so the work required to go straight from A to B is equal to that required to go through a bunch of conductors before coming back to the battery. That's why battery strengths are given in "Volts."

We can then ask how much energy is required to make an elementary charge - $Q = 1.6 \times 10^{-19}\mathrm{C}$ - move through a potential difference of 1V. This unit of energy - $1.6 \times 10^{-19}\mathrm{J}$ - is called the "electron-Volt", denoted eV. It's a standard measure of energy whenever you deal with fundamental particles, such as in particle beam cancer therapies. Molecular, chemical, atomic, and nuclear studies benefit from this more convenient unit of energy (the Joule is good for human-scale applications).

We often use "voltage" to mean "potential difference." This is *especially* true in electric circuits. We'll see later when we discuss magnetism that there is a key, subtle difference between the two.

Here are some typical potential differences in nature:

Between the human arm and leg during the heart's electrical activity	1 mV
Across biological cell membranes	80 mV
Between the terminals of a flashlight battery	1.5 V
Car battery	12 V
Electric Outlet (U.S.)	110V
Between long-distance electric transmission line and the ground	365 kV
Between base of thunderstorm cloud and the ground	100 MV

It only takes a few MV to ionize air - convert it into a plasma gas. Lightning is just such a phenomenon, and you can see from the potential above why lightning is so easy for thunderstorms.

Curved Paths and *Non-Uniform* Fields: Generalized Potential Difference

If the electric field isn't uniform or the path is not straight, then we need to generalize our formula for potential difference. In that case, we break the path into tiny little pieces, $d\vec{r}$, and evaluate the electric potential difference for each piece:

$$dV = -\vec{E} \cdot d\vec{r}$$

The total potential difference is then the sum of infinitesimal pieces:

$$V_{AB} = \int dV = -\int \vec{E} \cdot d\vec{r}.$$

This is the most general computation of potential difference that we can write down.

The potential of a point charge

Let's consider our old friend, the point charge. We know how to write the electric field for such an object:

$$\vec{E}_{point} = \frac{kq}{r^2}\hat{r}$$

Let's find the potential difference between two points A and B that lie along a radial line pointing outward from the charge. One point is a distance r_A from the charge, the other r_B. Because the field is *NON-UNIFORM*, we can't just multiply a field strength times the displacement. We have to use our general formula:

$$dV = -\vec{E} \cdot d\vec{r} = -\frac{kq}{r^2}\left(\hat{r} \cdot (\hat{r}dr)\right) = -\frac{kq\,dr}{r^2}$$

Now we just need to integrate from the point r_A to the point r_B:

$$V_{AB} = -\int_{r_A}^{r_B} \frac{kq\,dr}{r^2} = -kq\left[-\frac{1}{r}\right]\Bigg|_{r_A}^{r_B} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right).$$

Does this make sense?

- For $q > 0$ and $r_B > r_A$, the potential difference is negative and a positive test charge at r_A would move toward a r_B, "falling" as it were in the potential. It would take work to move the test charge back to r_A, fighting the repulsion between it and the source charge.
- Our result holds well if we change the source charge to $q < 0$, as the

sign of the charge flips and so do all the conclusions.

Since the path doesn't matter, the above result holds for ANY two points A and B around a point charge q. They don't have to lie on the same radial line.

The Zero of Potential

Only potential differences have meaning. We can conveniently choose a "zero potential" in a problem, or in life, to act as a surrogate zero point. This makes it clear, then, to what we are making relative potential measurements. For instance, in home wiring it's convenient to pick the earth ("ground") as the point of zero potential. In a car, it's convenient to define the body of the case as zero potential.

What about for a point charge? We see that as $r_A \to \infty$ above, $V \to 0$. In problems involving point charges (or charged spheres), it's convenient to define zero potential at infinity.

It shouldn't bug you that as you go an infinite distance away, the potential remains finite. Since the field falls off as you go further away, the work required to move the charge decreases as well, and the work required to move the charge remains finite. This is the same thing that happens with the earth and rocketry - there is an "escape velocity" which would carry an object launched from the earth out to infinity, escaping the earth's gravitational attraction.