General Physics - E&M (PHY 1308) Lecture Notes

Lecture 010: Solving Problems Involving Work and Potential

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no tags

Goals of this Lecture

- Setup and work problems involving potential, energy, etc.

Clarification from last time: the meaning of the "Volt"

I was a little careless in my definition of the Volt in last lecture. To be absolutely clear:

- The unit "Volt" tells you how much work is being done on every Coulomb of charge.
- To say that a battery is a 12V battery is to say that is does 12J of work on every Coulomb
- More volts means more work can be done on the same amount of charge.

The Boston Science Museum Van de Graaff Generator

We've watching video of the person in the cage being subjected to huge bolts of electricity from the large Van de Graaff generator at the Boston Museum of Science. Let's use this as a problem we can setup and work.

The Van de Graaff consists of a conductive ball at the top which is slowly imparted a large electric charge.Electrons are being stripped off the sphere by a belt, so the sphere acquires a positive charge. Being a conductor, all the charge migrates to the outer surface of the sphere, building up on the surface.

The *VdG* at the museum has a radius of 2.30m and develops a charge of $Q = 640\mu\mathrm{C}$. Considering this device to be a single, isolated sphere of charge, let's:

(a) find the potential at its surface,
(b) find the work needed to bring a proton from infinity to the surface of the sphere, and
(c) find the potential difference between the sphere are a point that is $2R$ from its surface (basically, the location of the cage protecting the person).

(a) We've worked problems involving spheres of charge before. As you saw on your homework (the slab problem), as long as you're outside the surface of the charged sphere the electric field is the same whether the charge is spread throughout the volume or concentrated on the sphere. For the case of a sphere, the electric field is the same as that of a point charge carrying Q located at the center of the sphere:

$$\vec{E}_{outside} = \frac{kQ}{r^2}\hat{r}$$

Now we can evaluate the potential at the surface of the *VdG* generator sphere. But remember, **it is meaningless to speak only of the potential as an absolute thing; it's a relative thing, and we have to define the "zero" of potential in order to compute the potential difference.**

For a function that falls off AT LEAST AS FAST AS $1/r^2$, we see that there is one place where the field strength goes to zero: at infinity. We can therefore choose to define *zero potential for a point charge, or our sphere, as existing at* $r = \infty$. It may seem strange to do this - that the potential can be finite at infinity - but keep in mind that since the field falls off as $1/r^2$ the work required to move a charge further from the sphere becomes less and less, so we have a finite result in the end.

So, choosing the location with respect to which we measure the potential difference as $r = \infty$, we can then calculate the potential at the surface of the sphere:

$$\Delta V_{AB} = V_B - V_A = V(R) - V(\infty) = -\int_R^\infty \vec{E} \cdot d\vec{r}$$

We need to figure out $d\vec{r}$. If we choose our path from R to ∞ as lying along a radial vector pointing outward from the sphere, then $d\vec{r} = dr\,\hat{r}$, and $\vec{E} \cdot d\vec{r} = Edr$. Substituting for E:

$$V(R) - V(\infty) = -\int_{R}^{\infty} \frac{kQ}{r^2} dr = -kQ\ ()|_{R}^{\infty} = \frac{kQ}{R} \approx 2.50\text{MV}.$$

Does this make sense? From last time, I mentioned that air breaks down (allowing things like lightning to happen) at a few MV, so this sounds like something that could make some wicked sparks.

(b) So how much work does it take to bring a proton ($q = 1.6 \times 10^{-19}\text{C}$) from $r = \infty$ to $r = R$? Well,

$$V(R) = -W_{R,\infty}/q$$

So

$$W_{R,\infty} = -qV(R) = 4.0 \times 10^{-13}\text{J} = 2.5\text{MeV}$$

(c) We now have the potential difference between infinity and the surface of the sphere at R. What about between R and 2R? The problem is nearly the same:

$$V(2R) - V(R) = \frac{kQ}{2R} - \frac{kQ}{R} = -\frac{kQ}{2R} = -1.25\text{MV}$$

Does this make sense? Why is this voltage negative while the voltage of R with respect to infinity is positive?

Draw a diagram of $V(r)$ with respect to $V(\infty)$. It will fall as $1/r$ for $r > R$ and be flat (fixed at kQ/R) for $r < R$. Why flat for $r < R$? Since the electric field is zero inside the sphere, it takes no more work to move a charge to $r < R$ than it did to move it to $r = R$.

We see that $V(2R)$ is less than $V(R)$. Thus, we expect $V(2R) - V(R)$ to be negative. On the other hand, we expect $V(R) - V(\infty)$ to be positive.

This problem illustrates a few things:

- Be careful with your signs. Get your definitions right, make sure you carry through the signs of your electric charges, and you'll be steered on the right path by the math
- Having been exposed to charged spheres and other "simple" geometries, you have a powerful toolkit to begin to understand the world around you.

Potential Difference: a high-voltage power line

Let's consider a long, straight power line with radius $1\mathrm{cm}$. It carries a linear charge density $\lambda = 2.6\mu\mathrm{C/m}$. Assuming no other charges are present, what's the potential difference between the wire and the ground. The ground is 22m below the wire.

Let's treat the wire as an infinitely long charge distribution with line symmetry. We've looked at that before, both from Coulomb's Law and from Gauss's Law. Recall that the electric field surrounding such a wire looks like:

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

Again, we need to think about what's asked. Point A in this problem is a point on the wire, and point B in this problem is a point on the ground. Let's choose the path between these two points to be a straight line, along a radius pointing outward from the wire, in a direction $\hat{r}$. So, let's setup the problem:

$$\Delta V_{AB} = V(r_B) - V(r_A) = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{\lambda}{2\pi\epsilon_0 r} dr(\hat{r}\cdot\hat{r}) = -\frac{\lambda}{2\pi\epsilon_0}\int_{r_A}^{r_B}\frac{1}{r}dr$$

That integral yields

$$\int_{r_A}^{r_B} \frac{1}{r} dr = ln(r)|_{r_A}^{r_B} = ln(r_B) - ln(r_A)$$

Plugging this in:

$$\Delta V_{AB} = -\frac{\lambda}{2\pi\epsilon_0}\left(ln(r_B) - ln(r_A)\right) = \frac{\lambda}{2\pi\epsilon_0}\left(ln(r_A) - ln(r_B)\right) = \frac{\lambda}{2\pi\epsilon_0} ln\left(\frac{r_A}{r_B}\right)$$

Plugging in our numbers:

Unknown control sequence '\kV'

As I mentioned in the last class, this is a typical electric potential difference for a high-voltage power line and the ground.

This problem illustrates a few things:

- You can't always define zero potential at infinity. If you set $r_B = \infty$, the logarithm blows up ($ln(1/\infty) = ln(0) = -\infty$). Why is this? Physically, an infinite line of charge will NEVER look like a point charge, no matter how far away you get. You also see that $1/r^2$ is the minimum power of 1/r where you can use infinity as a zero point for potential.
- You can't avoid the integral. Get comfy with it. It's going to be a critical tool throughout this class.

Finding the total potential superposition of potentials

What happens when we *don't* know the field of a charge distribution, or when the field is just too complicated to integrate easily? In that case, we can use superposition. This often provides an easier approach to calculating the field as well.

Given an unknown electric field, you can treat the source of the field each as a tiny point source of electric field. If we do that, then we know that the potential associated with each point is:

{ V(P) = sum_i frac{kq_i}{r_i}]

since the electric field from each point is just $\vec{E} = (kq_i/r^2)\hat{r}$. The r_i are the distance from each point charge to the location P.

Why is this useful? This formula has an ENORMOUS advantage over $\vec{E} = (kq_i/r^2)\hat{r}$ - it's a scalar, a number. It has no direction associated with it. Numbers are easier to add than vectors (but, that's no excuse!).

Electric potential, $V(P)$, is a scalar - it has no direction. No need to compute angles, vector components, or unit vectors.

Let's go integral with this. Imagine now we subdivide the charge distribution into a huge number of infinitesimal, equal-sized point charges. Now we need to integrate over the potential due to each point charge to get the total potential:

$$V(r) = \int dV = \int \frac{kdq}{r}$$

That's much simpler looking. It won't allow you to calculate the vector part of the electric field, but it WILL allow you to get the magnitude. It turns out we CAN get the direction, too, but that's a topic for the next lecture.