General Physics - E&M (PHY 1308) Lecture Notes

Lecture 012: Electrostatic Energy and Capacitors

SteveSekula, 21 September 2010 (created 15 September 2010)

no tags

Goals of this Lecture:

- Introduce the idea of energy stored in the electric field (electrostatic energy)
- Introduce the idea of a "capacitor", capacitance, and discuss simple capacitors

Electrostatic energy

The electric field can not only do work, but store energy. The storage of energy in electric fields, and its release, is the basis of chemical energy. The metabolizing food and burning fuel are basically the act of rearranging configurations of electric charge so as to release some of the energy stored in the original arrangement. The example we'll start with today seems simple, but it's deceptively powerful because it contains the essential elements of any system that stores energy in the electric field.

This exercise will not only introduce the idea of energy stored in an electric field, but will help to review some of the basic concepts we started learning a few weeks ago.

Imagine that I place a point charge, $q_1 > 0$, in a region of empty space free from any electric fields or other forces. How much work does it take to place q_1 at a point in such a region free of electric fields and other forces?

- ANSWER: none. Absent any source of external force, we can place charge q_1 at a point at no cost in energy. Remember that $W_{AB} = \int \vec{F} \cdot d\vec{r}$ - if $\vec{F} = 0$, then no work is needed.

Now, let's imagine that I want to bring a second positive charge, q_2, from very far away up to a distance $|r| = a$ from q_1. I want to figure out the work required to do that. Let's attack this from the perspective of electric potential difference:

$$\Delta V_{AB} = V_B - V_A$$

If we are going to bring another charge q_2 from infiniity to a distance a from q_1, then:

$$\Delta V_{r,\infty} = -kq_1/r$$

So the work required to bring that charge to a point a from q_1 is:

$$W_{AB} \equiv W_2 = -q_2 V(a) = kq_1 q_2/a$$

Now we bring in a third positive charge, q_3, also from very far away (infinity). We have to do work to get it there, against the repulsion of the other two charges. If we also place the third charge a distance a from each of the other two, forming an equilateral triangle, then we need to do work:

$$W_3 = kq_1 q_3/a + kq_2 q_3/a$$

So the total work required to form this configuration of charges is:

$$W = W_1 + W_2 + W_3 = 0 + kq_1 q_2/a + kq_1 q_3/a + kq_2 q_3/a$$

Because electric forces are conservative forces, the work done to make the configuration is equal to the energy stored in the electric field. It takes energy to hold the charges together, to keep them from flying apart; that means energy is stored in the configuration.

Releasing any of the charges converts stored energy in the field into kinetic energy in one or more of the electric charges. Energy stored in an electric field can be released in other forms of energy, such as kinetic energy.

It doesn't matter in what order we assemble this simple example; the energy stored is the same. If one of them had been negative, it would *take work to separate that charge from the other two due to its attraction.* Electrostatic energy can be positive or negative, depending on whether or not it took work to assemble the charges in the first place.

This example is a simple metaphor for a molecule, such as a water molecule. In fact, in water the electrostatic energy is negative and it takes an injection of work (costs energy) to separate the hydrogen from the oxygen. For water, electrostatic energy is NEGATIVE. Equivalently, that is the energy released when water forms from individual atoms.

Capacitors

So we can store energy in an electric field. A device that does this is called a *capacitor*. Specifically,

- A **capacitor** is a pair of electrical conductors that carry equal but opposite charges

The two conductors are thus attracted to one another, and it takes work to keep them apart. The easiest to analyze capactor is the **parallel-plate capacitor**, although capacitors come in many configurations. Understanding the parallel-plate capacitor will give us insight into electrostatic energy and the electric field.

Parallel plate capacitor

A device with two parallel thin sheets of conductor. Charge is removed from one plate and added to the other (e.g. a battery can do this). Thus we have equal but opposite charges on the two plates, and close to the center of the plates we can understand the electric field by modeling the system with two infinite thin sheets of opposite charge. The field lines are perpendicular to the sheets and go from positive to negative.

Closer to the ends, the field becomes nonuniform. But we can neglect this and still get tremendously far in understanding these devices.

If you do the Gaussian Surface trick and apply Gauss's Law to either of the plates, you'll find that the electric field at the surface of the conductor is given by $|E| = \sigma/\epsilon_0$. If we've spread out charge uniformly over the two plates, then for either plate $\sigma = Q/A$, where A is the area of the plate. Thus the uniform field between the two is:

$$E = Q/\epsilon_0 A$$

Do a demonstration of this using two parallel plates and a Gaussian Surface. Analyze the electric field between two parallel plates and show that it is $E = \sigma/\epsilon_0$ and NOT $E = 2\sigma/\epsilon_0$. Remember, these are conductors and the opposite charges accumulate on the faces closest between the two plates.

The potential difference between the two plates is

$$V = Ed = Qd/\epsilon_0 A$$

since the field is uniform.