General Physics - E&M (PHY 1308) Lecture Notes

Lecture 013: Energy Storage and the Application of Capacitors

SteveSekula, 22 September 2010 (created 21 September 2010)

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Goals of this Lecture:

- Continue the discussion of the parallel plate capacitor
- Define energy stored in a capacitor
- Learn how to handle systems of multiple capacitors, in series and in parallel

Parallel plate capacitor

A device with two parallel thin sheets of conductor. Charge is removed from one plate and added to the other (e.g. a battery can do this). Thus we have equal but opposite charges on the two plates, and close to the center of the plates we can understand the electric field by modeling the system with two infinite thin sheets of opposite charge. The field lines are perpendicular to the sheets and go from positive to negative.

Closer to the ends, the field becomes nonuniform. But we can neglect this and still get tremendously far in understanding these devices.

If you do the Gaussian Surface trick and apply Gauss's Law to either of the plates, you'll find that the electric field at the surface of the conductor is given by $|E| = \sigma/\epsilon_0$. If we've spread out charge uniformly over the two plates, then for either plate $\sigma = Q/A$, where A is the area of the plate. Thus the uniform field between the two is:

$$E = Q/\epsilon_0 A$$

Do a demonstration of this using two parallel plates and a Gaussian

Surface. Analyze the electric field between two parallel plates and show that it is $E = \sigma/\epsilon_0$ and NOT $E = 2\sigma/\epsilon_0$. Remember, these are conductors and the opposite charges accumulate on the faces closest between the two plates.

I can then ask how much work it takes to take a charge from one plate (point A) and move it through the electric field to the other plate (point B). That work is related to the *potential difference* between the plates. The electric field is uniform between the plates and doesn't depend on the location along the path. The potential difference between the two plates is then:

$$V = Ed = Qd/\epsilon_0 A$$

since the field is uniform.

We now have the essential ingredients to discuss the properties of and uses of capacitors.

Capacitance

We can rewrite our relationship between charge and potential to solve for charge in terms of potential:

$$Q = (\epsilon_0 A/d)V$$

- Charge is linearly proportional to the potential difference
- the proportionality depends on the geometry of the capacitor and the constant ϵ_0.

This geometric factor is called the **CAPACITANCE** of a capacitor, and basically tells you how the geometry contributes to the amount of charge accumulated per volt of potential difference applied:

$$C \equiv \epsilon_0 A/d = Q/V$$

For *any* capacitor, we have:

$$Q = CV$$

and specifically for the parallel plate capacitor we have the above. You can imagine, then, using potential difference and charge to determine the capacitance of a difference geometry, something other than a parallel plate system.

The units of capacitance are Coulombs/Volt, known as the "Farad" (F) in honor of Michael Faraday, who introduced the concept of an electric field - of a force per unit electric charge.

The Farad is a LARGE unit of capacitance. Typical capacitors you find in electronics are measured in micro-Farads, μF, or even pico-Farads (pF). Releasing the energy of a 1F capacitor - say, but shorting across the leads of the capacitor with a screwdriver - is sufficient to weld the metal of the screwdriver to the leads of the capacitor.

Energy Storage in Capacitors

How much energy can a capactor hold, given its capacitance and the electric potential difference to which it's subjected (e.g. by a battery)?

Imagine moving a small piece of charge from one plate to the other when there is a potential difference V between the plates. Let's write that small piece of charge dQ. The amount of work required to do that is

$$dW = VdQ$$

since potential difference is work per unit charge and the work here is the work done against the field.

By moving charge dQ from one plate to the other, I increase the electric field between the plates and thus increase the potential difference between the plates. From the definitions of capacitance (which doesn't change unless the geometry changes), that change in potential difference is then:

$$dQ = CdV$$

So the work involved in moving the charge is then:

$$dW = CVdV$$

Now, let's figure out what the TOTAL work involved in moving all of that charge from one plate to the other will be. We find that by adding up al the dW:

$$W = \int dW = \int_0^V CV\, dV = \frac{1}{2}CV^2$$

If this is the work required to move all that charge, then it's also equal to the energy stored in the electric field. Thus the electrostatic energy stored in a capacitor is:

$$U = \frac{1}{2}CV^2$$

Note: when we speak of a capacitor being "charged" or "charged up", we mean the charge on either of the plates. The net charge in the system remains zero.

Using capacitors

Capacitors can either have nothing (vacuum) between the plates, or there can be something (a non-conductive material) between the plates. If there is vacuum (nothing), then the capacitance is what we wrote above:

$$C = \frac{\epsilon_0 A}{d}$$

If instead we stick material between the plates called a **dielectric**, we can increase the capacitance of the device. Dielectric materials are electrically neutral, but made from very many tiny dipoles. When subjected to an electric field, the dipoles align with the field and weaken it overall. When the strength of E decreases for the same Q as before, then V decreases too. This makes capacitance, $C = Q/V$, go up.

The amount that the capacitance increases over the case without a dielectric is called κ, the **dielectric constant**. When a capacitor contains a dielectric, the capacitance is given by:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

Most materials have a dielectric constant between 2-10. Air is about $\kappa = 1.0006$, while glass is $\kappa = 5.6$ and water is $\kappa = 80$.

Connecting capacitors

Often, as in an electronic circuit, you will want to have more than one capacitor. How do you handle those situations where multiple/many capacitors are present?

Capacitors in Parallel

The easiest case to think about and work out is when two capacitors are *in parallel* - that is, when both capacitors are arranged in such a way that they experience the same potential difference, V. Draw such a circuit on the board. The capacitance of the two capacitors is C_1 and C_2.

Conducting wires connect the two sides of the capacitors to "ground" (zero voltage) and V (positive voltage). Let's think about the potential on just one side of the system. Since the wires of the circuit and the plates of the capacitors are all conductors, there are NO ELECTRIC FIELDS until we get to the gap between the plates. So a charge moving from the 0V side toward the plates of the capacitor feels no change in potential:

$$\Delta V_{AB} = \int \vec{E} \cdot d\vec{r} = 0$$

That is, until it reaches the gap between the plates, where the IS a uniform electric field. Here, we know that $\Delta V_{AB} = Ed$, reaching the final potential of V on the other side. Then the charge would enter more conductor, where there are no electric fields. Thus the is no more change in potential after crossing the gap.

This tells us that both capacitors feel the same potential difference when they are side-by-side (in parallel). It's like they are one big capacitor. Can we find a way to write the effective capacitance of the two capacitors, treating them as if they are one capacitor? If we could do that, then we'd just have one number to carry around with us - one capacitance.

Since V is the same for both, we have two equations relating charge and capacitance:

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

What if we wanted to find the total capacitance of this circuit, C_{total}? Well, the total charge on the system is just the sum of the charges (charges always add!):

$$Q_{total} = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2)V$$

So:

$$C_{total} = C_1 + C_2$$

In fact, if have have a whole bunch of capacitors all in parallel with one another, then the total capacitance is just the sum of the individual capacitances. That's it!

$$C_{total} = \sum_{i=1}^{N} C_i \text{(capacitors in parallel)}$$

You can understand this physically by thinking about parallel plates with equal spacing placed together in parallel. Essentially, you are taking individual capacitors and adding the areas of their plates together. That's what connecting them with conductors in a parallel does. That's equivalent to just adding their capacitances.

Capacitors in Series

What if, instead of putting capacitors side-by-side and connecting their different sides to different potentials, I places them in sequence. This is called "series" - the backend of one capacitor is connected to the frontend of the other, and so on. Draw this on the board.

The voltages are now no longer the same across the difference capacitors. What is the same? Draw the charges on the plates of one, and discuss how the charges on the plates of the other must be equal but opposite. In other words, $Q_1 = Q_2 = Q$ for two capacitors in series. We then have two equations:

$$Q = C_1 V_1$$

$$Q = C_2 V_2$$

We want to find the total capacitance. Since we have the same charge on both, we just have to sum the voltages this time. Voltages, like charges, add:

$$V_{total} = V_1 + V_2 = Q/C_1 + Q/C_2 = Q(\frac{1}{C_1} + \frac{1}{C_2})$$

$$Q = \frac{V_{total}}{(\frac{1}{C_1} + \frac{1}{C_2})}$$

From this, we realize that for capacitors in SERIES:

$$1/C_{total} = 1/C_1 + 1/C_2$$

So for capacitors in series, you have:

$$1/C_{total} = \sum_{i=1}^{N} \frac{1}{C_i}$$

The *combined capacitance is less than the individual capacitances*. Again, to think about the physics of this, take two parallel plate capacitors of equal area and plate separation. Put them in series. Now shrink the distance between them. We see that this is equivalent to a single capacitor with TWICE the spacing of the plates. Thus the total capacitance is less than the individual ones.

Energy in the electric field

Where, exactly, is the energy stored in a capacitor actually stored? The difference between a charged and uncharged capacitor lies in the arrangement of charge, which creates an electric field.

Well, what changes as we charge up the plates of the capacitor? The strength of the electric field is changing. There is no electric field in an uncharged capacitor. The size of the electric field must relate to the energy stored in the system.

Every electric field represents stored energy.

Allow the charges to recombine and you release stored energy - the field weakens. It is useful to define the **energy density** stored in the field. Since the amount of energy can vary with location in the field, defining the energy stored per unit volume is handy.

For a capacitor, we know that $U = 1/2CV^2$. For a parallel plate capactor, this yields:

$$U = 1/2CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}V^2$$

The volume of space between the plates of the capacitor is just Ad, and the potential is given by $V = Ed$. Thus the energy density can be written ONLY in terms of the electric field:

$$u_E \equiv \frac{U}{Volume} = \frac{1}{2}\frac{\epsilon_0 A}{d}\frac{1}{Ad}E^2 d^2 = \frac{1}{2}\epsilon_0 E^2$$

You are looking at something quite profound. This equation is universal, not only applicable to parallel plate capacitors. That means that any electric field, regardless of its origin, contains this energy density. Changing the field releases energy. It is this process, in fact, that underlies so much of the energy that drives chemical and biological systems. This same equation, too, will lead is to a fundamental understanding of light.