General Physics - E&M (PHY 1308) Lecture Notes

Lecture 014: Electric Current

SteveSekula, 27 September 2010 (created 26 September 2010)

no tags

Goals of this lecture

- Motivate the importance of the motion of electrons
- Describe the motion
- Discuss resistance to the motion (Ohm's Law)
- Discuss the work done by electric current

Review of capacitors and potential

The establishment of an electric potential difference, ΔV_{AB}, as through the creation of an electric field (e.g. in a battery) allows work to be done on electric charge.

Capacitors are devices that store energy in an electric field through spatial separation of charges (e.g. work is done on the charges to distribute the positive charges on one side and the negative on the other. A **charged capacitor** represents an *electrostatic equilibrium* situation, where no electric fields are present in the conductor material in the capacitor or in the leads connecting it to the battery.

We used the electrostatic equilibrium situation, and the addition of potentials, to figure out how to understand systems containing more than one capacitor in series or in parallel. We also defined capacitance as the geometric contribution of the device to energy storage, and we saw how it related to charge and voltage ($Q = CV$).

Today, we make things a bit more complicated by abandoning electrostatic equilibrium. To do that, we need to develop some new language.

Electric Current

We have so far considered electrostatic equilibrium - situations where charges are not moving (anymore). We've ignored their motion in between. Today, we concentrate on what happens WHEN charges move. Why? Moving charges and the work they do is the basis of electrical appliances, computers, and even the function of cells. To get at the causes underlying those benefits, we have to understand the movement of electric charge.

Let's think about charge moving through a conductor in the same way we think about water moving along a stream or through a pipe. We need to have a way of quantifying the amount of charge crossing a certain area per unit time. This is called *current*, and has the units of Coulombs/Seconds or "Amperes." We denote current by the symbol I.

If we are interested steady current, flow that is stable over time, or the time-average of current we can write:

$$I = \Delta Q/\Delta t$$

On the other hand, a more general notation takes into consideration that the flow can vary with time (e.g. as described by some complex function). Then we write:

$$I = dQ/dt$$

By convention, current is POSITIVE in the direction that positive charge is flowing. So for electrons in a metal, the current flows in the OPPOSITE direction of the electrons. A single current can consist of different charges doing different things. Just remember:

- NET CURRENT is the sum of the positive and negative currents
- If positive and negative charge move in the same direction, the net current is ZERO
- If positive and negative charge move in OPPOSITE directions, there is non-zero net charge

Use the DC Circuit Construction Kit to motivate a discussion of these concepts. Which way is the current flowing? http://phet.colorado.edu/en/simulation/circuit-construction-kit-dc-virtual-lab

A microscopic look at current

At the most fundamental level, electric current is the motion of fundamental charges under the influence of an external electric field (an electric potential difference). Electric current is, then, quite complicated, depending on the number of charge carriers, their density, and their charge.

Charges in a metal free from external electric fields are always in motion, but it's random thermal motion so there is no net current in any one direction. When we apply an electric field, through a battery, we induce a small "drift velocity" in one specific direction (for electrons, against the electric field in the battery) and this results in current.

Let's imagine this situation. We have a bunch of fundamental charge carriers with charge q and drift velocity v_d. We want to express the current in terms of microscopic properties (charge and speed) and macroscopic properties of the conductor (length, area). Let's denote the conductor's cross-section as A and length as L. The volume of the conductor is then $V_{conductor} = AL$.

If the number of charges per unit volume (number density) is n, then the number of total charges in a given volume of the conductor is nAL. Thus the total charge in this volume of the conductor is:

$$\Delta Q = nALq.$$

That's the numerator of current. What about the denominator? How long does it take them to pass a given point in the conductor? If they are moving at a speef v_d, then the time it takes them to move through the length of conductor under question is:

$$\Delta t = L/v_d.$$

The current is then:

$$I = \Delta Q/\Delta t = \frac{nALq}{L/v_d} = nAqv_d.$$

We can apply this to a copper wire with a cross-sectional area of 1mm. Copper has a number density of free charges of $n = 1.1 \times 10^{29}\text{m}^{-3}$. If a 5.0A current flows through the wire, what is the speed of the charges (electrons)?

$$v_d = \frac{I}{nAq} \approx 0.28\text{mm/s}.$$

That's not fast at all, but it's actually correct. Why does a light come on the minute you flip a switch, then? The answer is because the electric field, which gives charges their marching orders, is established nearly instantaneously (at the speed of light, approximately) within the copper, All charges start moving right away, albeit at the above speed. They do work right away, even though they're moving at 0.28mm/s.

This example helps us make a distinction between the speed of the electrons and the speed of electric fields (electric signals). Drift speeds are actually kinda slow, while signals (fields) establish and travel far, far faster (speed of light). There is almost no time delay between establishing the field and the start of movement of the charges themselves.

Current Density

What if electric current is occurring in a more complex medium (e.g. NOT in a wire)? What about through a cell membrane, which isn't simply describable as a single wire carrying ions into and out of the cell. In that case, we need a more general quantity: current density, the amount of current flowing per unit area of the path.

Current density is by definition a vector, whose direction is given by the direction of current flow at a given point and whose magnitude is the current per unit area. Dividing out microscopic current equation by the area of the conductor:

$$\vec{J} = nq\vec{v}_d.$$

We see the product of drift velocity direction and charge defines the overall direction (n is always positive). So electron current densities point opposite the velocity of electrons, as per our convention that current is in the direction of positive charge motion.

As an exercise, consider an ion pump in a cell membrane. What is an ion pump? It's a protein that is capable of allowing positive ions into or out of the cell itself. That protein is embedded in the cell membrane. We can analyze the current and current density when a channel opens for 1ms and allows 1.1×10^4 singly-ionized potassium ions through ($q = +e$). The radius of the channel, treating it like a little cylinder, is $r = 0.15\text{mm}$.

- What is the current? ANSWER: $I = \Delta Q/\Delta t = 1.8\text{pA}$
- What is the current density? ANSWER: $J = I/A = 2.5 \times 10^7 \text{A}/\text{m}^2$