General Physics - E&M (PHY 1308) Lecture Notes

Lecture 017: Electric Circuits - Resistors and Batteries

SteveSekula, 4 October 2010 (created 3 October 2010)

no tags

Goals of this lecture

- Develop the mathematics to describe what we saw last time
- Introduce the language of circuits

Review from last time

We hooked up lightbulbs - resistors - in parallel and series to a 110V electric potential difference. We made a few observations:

- resistors (lightbulbs) in parallel achieve their full power (same voltage, different currents)
- resistors in series don't achieve their full power (different voltages, same current).

Today we can develop the language to describe what we saw.

Series and Parallel Resistors

- Introduce the symbol for the resistor and use this to discuss a basic circuit diagram
  - Lines represent very, very good conductors (assumed to be perfect)
  - jagged squiggles describes resistors - conductors whose resistivity and geometry inhibit the flow of electrons
  - for a bit longer, we'll denote the source of electric potential difference as a gap with a "V" inside the gap.

We considered series and parallel capacitors. Series and parallel are the two simplest way to connect ANY two electronic components. Two components are in series if the current has nowhere to go but through both components. They are in parallel if the current can flow *either* through one *or* the other component.

Let's consider series and parallel resistors.

Series Resistors

When in series, and if the circuit is in a *steady state* - that is, there is no charge buildup anywhere in the circuit - then the current has no choice but to go through one resistor, R_1, and then the other, R_2.

The current through series resistors is the same in both resistors

We can write Ohm's Law for each resistor:

$$V_1 = I_1 R_1$$

$$V_2 = I_2 R_2$$

We know that $I_1 = I_2 = I_{total}$. We also know that the total electric potential in the circuit is V_{total}, and that the sum of the individual potential differences must add up to that:

$$V_{total} = V_1 + V_2$$

Finally, we know that whatever the TOTAL resistance of the circuit, there is also an Ohm's Law for that:

$$V_{total} = I_{total} R_{total}$$

We can then solve for the total resistance:

$$V_{total} = V_1 + V_2 = I_{total}(R_1 + R_2) = I_{total} R_{total}$$

Thus we can make the identification that:

$$R_{total} = R_1 + R_2.$$

The total resistance of series resistors is the sum of the individual

resistances.

We can then reverse this and use Ohm's Law to solve for the voltage across each resistor:

$$V_1 = \frac{R_1}{R_1 + R_2} V_{total}$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_{total}$$

Add these up and you will recover the total voltage, V_{total}.

Ideal and Real Batteries

Let's take a moment to talk about batteries. We've been using an unnamed "source of electric potential difference, V" to attack these problems. An *ideal battery* delivers a steady voltage across its terminals, regardless of the current through the circuit. We denote the ability of a battery to create an electric field and do work on charge as its *electromotive force*, or EMF. We write the work per unit charge that the battery can do as $\mathcal{E}$.

Let's revisit series resistor results in lieu of this change in notation, now that we're introducing real batteries:

$$V_1 = \frac{R_1}{R_1 + R_2} \mathcal{E}$$

$$V_2 = \frac{R_2}{R_1 + R_2} \mathcal{E}$$

The battery voltage divides between the two resistors, based on their resistance. That is also why a combination of series resistors is called a **voltage divider**

- If you need to take a large source of EMF and subdivide its voltage into smaller pieces to work on more delicate equipment, you need to use series resistors to create a voltage divider.

- Consider the kindle. In your homework, you determined that the voltage required to move *E-Ink* particles is really tiny - about 0.2mV. Why can I plug my kindle into the wall and not be afraid of the high voltage - 110V - frying my electronics? I'm protected by at least one voltage divider built into the circuitry, tamping down the wall voltage to something more reasonable for this device.

Let's think about *real batteries* for a second.

What differentiates AAA, AA, and D-type 1.2V batteries? They all deliver 1.2V of potential difference, so why the differentiation? (Is this just a conspiracy to force me to buy different batteries for all my toys?!)

Use this discussion to motivate the discovery that different battery types can deliver different currents in the same period. D-type batteries can deliver about 3A for an hour, *AA-type* batteries 2.5A for an hour, and *AAA-type* batteries about 1A for an hour. That must mean that their internal RESISTANCES are different. According to the Energizer company's own data, *AA-type* batteries have an internal resistance of about 0.2Ω.

http://data.energizer.com/PDFs/BatteryIR.pdf

So batteries are sources of electric potential difference AND resistance. The chemical reactions that allow them to create an EMF are not steady over time (batteries wear down, and batteries can't behave perfectly for all currents). In effect, a battery is a source of EMF in series with a resistor, creating an internal voltage divider. The terminal voltage is always a bit less than the rated voltage for the battery. If the resistive load in the external circuit, R_L, is big compared to the internal battery resistance, R_{int}, then the voltage delivered by the battery is nearly $\mathcal{E}$:

$$V_{battery} = \frac{R_L}{R_{int} + R_L}\mathcal{E} \rightarrow R_{int} << R_L \rightarrow \mathcal{E}$$

Even if we short-circuit the battery (connect its terminals - ALWAYS A BAD

IDEA!), we don't get infinite current because of that internal resistance. In fact, the most current we ever get is

$$I_{short} = \mathcal{E}/R_{int}$$

For a *AA-type* battery, that gives us:

$$I_{short} = (1.2\text{V})/(0.2\Omega) = 6A$$

How much power is that? Will that be bad?

Unknown control sequence '\W'

That's enough to raise the temperature of your hand, in contact with the battery, by 1-2 degrees celcius each second. That would be noticeably hot in a very short period of time, and it's not at all good for the battery.

Parallel Resistors

What about resistors in parallel? Now, the current across the two resistors is not the same, but the potential difference IS the same. Thus:

$$I_{total} = I_1 + I_2$$

$$V_{total} = V_1 = V_2$$

We can then solve for the total resistance of parallel resistors:

$$I_{total} = V_{total}/R_{total} = I_1 + I_2 = V_1/R_1 + V_2/R_2 = V_{total}\left(1/R_1 + 1/R_2\right)$$

Thus:

$$1/R_{total} = 1/R_2 + 1/R_2$$

And the total resistance is given by:

$$R_{total} = \frac{R_1 R_2}{R_1 + R_2}$$

Note that series capacitors combine in the same way as parallel resistors, and parallel capacitors combine in the same was as series resistors. The path of more resistance carries less current than that of least resistance:

$$I_1 = \mathcal{E}/R_1$$

$$I_2 = \mathcal{E}/R_2$$

If $R_1 > R_2$, then $I_2 > I_1$ from Ohm's Law. Current flows more easily along the path of least resistance.

Analyzing Circuits

Let's discuss strategies for analyzing circuits.

Work example 25.3 in Wolfson

Important: Ohm's Law relates voltage *across a resistor* to current *across a resistor*, not arbitrary voltages and currents anywhere in a circuit. Just because there is 12V across a circuit DOESN'T mean there are 12V across each component. Apply Ohm's Law one component at a time, and combine components where possible.