General Physics - E&M (PHY 1308) Lecture Notes

Lecture 018: Kirchoff's Laws

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no tags

Goals of this Lecture

- Motivate and introduce Kirchoff's Laws

Kirchoff's Laws

Some circuits cannot be simplified (e.g. by combining parallel and series resistors into a single resistor). Consider the diagram in Fig. 25.13 in Wolfson. That circuit cannot be analyzed into simple series and parallel combinations. How do we tackle it?

Fundamentally, we need to apply conservation laws. The number of charges going into a part of the circuit must come out the other end, otherwise charge will build up in between and we cannot be in a steady-state situation. Kirchoff's Laws are just the realization of the application of conservation laws to the system.

Kirchoff's Laws analyze circuits in terms of *loops* - closed paths in the circuit. Why? Because if you traverse a closed path and measure things, like current, as you go, you have to return to where you started. In other words, the sum of all the changes in the energy per unit charge have to sum to zero. That is, increases and decreases in voltage must sum to zero, otherwise you're not returning to where you started.

Kirchoff's Loop Law: the sum of voltages around a closed loop is zero. This is essentially a restatement of the conservation of energy.

- The loop law holds for any closed loop in a circuit, regardless of the complexity of the circuit. That's because fundamentally, nature always conserves energy.

In analyzing resistors in series, we observed that in steady-state the charge per unit time entering a series of resistors must be equal to the charge per unit time exiting the system. This is essentially the statement that the sum of the charges in a closed system is constant - *conservation of charge*. That means that if we consider a *node* in a circuit - a place where several conduction paths meet - the sum of the currents into the out of the node must sum to zero.

Kirchoff's Node Law: the sum of the currents at any node in a circuit is zero.

Strategies for applying Kirchoff's Laws

Let's apply these laws - essentially, conservation of energy and conservation of charge - to a multiloop circuit like the one in Example 25.4 in Wolfson. Our strategy will always be the same:

- Identify LOOPS and NODES. Label them. Do whatever you need to do to notate their existence.
- Label the currents at each node, assigning a direction to each. This is the trickiest part of the strategy, because often it's not clear where current is actually flowing and you have to think hard and carefully about your choices. Here are some rules you can follow to help you assign directions and signs:
  - For all but one node, write equations expressing Kirchoff's Node Law: the sum of the currents at any node is zero. Take currents flowing INTO the node as positive and OUT of the node as negative.
  - Write equations representing Kirchoff's Loop Law for as many independent loops as necessary. Make a decision as to whether you will go *clockwise* or *counterclockwise* around your loops, and BE CONSISTENT.
  - The voltage change going through a battery from the negative to the positive terminal is $+\mathcal{E}$; The voltage change from + to - is $-\mathcal{E}$.
  - For resistors traversed in your direction assigned to the current, the voltage change is $-IR$; for the opposite direction, it's $+IR$
  - For other circuit elements (e.g. capacitors), use the characteristics of the element to determine the voltage change. We'll come back to

that one in a bit.
  - You don't need equations for all loops and nodes; some will be redundant. Our example will illustrate this.
- Solve the equations for unknown quantities.
- Check your answer to see if it makes sense based on how you assigned directions.

Let's find the current in resistor R_3 in example 25.4. I choose to go counterclockwise in LOOP 1 and clockwise in LOOP 2 - that way, current in the center resistor is flowing in the same direction whether I look at LOOP 1 or LOOP 2. I accept that once I make this choice, I stick to it; even if current isn't REALLY flowing in that direction, if I stick to my definition I'll be OK.

Let's label the left loop at LOOP 1 and the right loop as LOOP 2, and the whole outer loop as LOOP 3. Let's label the top node A and the bottom node B.

- We only need to work the Node Law at one node. so let's choose node A:

$$0 = -I_1 + I_2 + I_3 \text{ (node A)}$$

- We need loop equations for all but one loop, since two of the three overlap. Let's start with LOOP 1. Beginning at Node A, we go counterclockwise. We first encounter an EMF, which is $+\mathcal{E}_1$ because we proceed from the negative to the positive terminal. We then encounter resistor 1, which gives us a potential change of $-I_1R_1$. We then encounter resistor 3, which gives us a potential change of $-I_3R_3$. Thus:

$$0 = +\mathcal{E}_1 - I_1R_1 - I_3R_3 \text{ (Loop 1)}$$

- We need another loop equation. Let's do LOOP 2, going CLOCKWISE in this loop and starting at node A. We encounter a battery first, which gives us a positive EMF $+\mathcal{E}_2$. We then encounter resistor 2, giving us a potential change of $+I_2R_2$ (because we're going AGAINST the current).

We then encounter resistor 3, giving us a potential change of $-I_3R_3$. Thus:

$$0 = +\mathcal{E}_2 + (I_2R_2) - I_3R_3 \text{ (Loop 2)}$$

Let's plug in numbers for the known quantities in these equations:

$$0 = -I_1 + I_2 + I_3 \text{ (node A)}$$

$$0 = +6 - 2I_1 - I_3 \text{ (Loop 1)}$$

$$0 = +9 + 4I_2 - I_3 \text{ (Loop 2)}$$

We want I_3. We can eliminate I_1 by using the node A equation:

$$I_1 = I_2 + I_3 \text{ (node A)}$$

Substituting into the loop 1 equation yields:

$$0 = 6 - 2I_2 - 3I_3 \rightarrow I_2 = \frac{1}{2}(6 - 3I_3) \text{ (Loop 1)}$$

Finally, we substitute that into the Loop 2 equation and remove I_2:

$$0 = 9 + 2(6 - 3I_3) - I_3 \rightarrow I_3 = 3\text{A} \text{ (Loop 2)}$$

We had assigned I_3 an algebraic sign that was positive, flowing INTO node A. The result above - a positive current for I_3 - confirms that we inadvertantly chose the correct sign. This is actually clear from the picture - since both batteries have negative terminals adjacent to node A, current must be flowing into node A from the bottom and out the top into the batteries. If one of the batteries were reversed, the situation wouldn't be so clear and we have to rely on our math to tell us the answer.

Measuring voltage and current

A *voltmeter* is a device that reads the voltage across a component in a circuit. It possesses a huge resistance; if it did not, then it would provide too attractive a path for current to flow and that would radically alter the voltage across the device being measured.

An *ammeter* measures current flow in a circuit, and is inserted into the path of the current in the circuit. It possess as SMALL a resistance as possible, to avoid altering the potentials downstream and thus altering the current.

Capacitors in Circuits

What happens if we add a capacitor in series with a resistor in a simple circuit? Now, for the time between activating the switch in the circuit and the time the capacitor is fully charged, current is NOT steady-state in the system; in fact, it's flowing but it's BUILDING UP on the capacitor. Eventually, this prohibits the further flow of charge in the system as the electric field in the capacitor is equal to but opposite the potential in the battery. At that point, the circuit is in ELECTROSTATIC EQUILIBRIUM and no current flows. If we then REMOVE the battery and connect the capactor across the resistor, current flows in the opposite direction as negative and positive charges rejoin each other. There is current flow at first, but then less and less later until all charges are pairs and current flow stops altogether again. This kind of circuit is known as an RC circuit. We will analyze it physically and mathematically, and draw conclusions about these systems.

Charging

Consider a battery-switch-resistor-capacitor circuit.

- Flip the switch and consider times very close to that event. Charge flows freely at first, negative charges pulled off one side the capacitor and placed onto the other.
- At first, the current through the resistor is $I = \mathcal{E}/R$. It decreases as the voltage across the capacitor increases (since $Q = CV$, as Q increases so

does V across the capacitor)
- Eventually the voltage across the capacitor increases to match that of the battery
- The whole system reaches a state where the current is zero and the capacitor is charged.