General Physics - E&M (PHY 1308) Lecture Notes

Lecture 019: RC Circuits

SteveSekula, 7 October 2010 (created 7 October 2010)

no tags

Goals of this Lecture

- Describe the behavior of RC circuits using Kirchoff's Laws

Charging a Capacitor: Kirchoff's Law View

We can analyze the circuit using Kirchoff's Loop Law. Let's go clockwise, and assume the current points in the same direction:

$$\mathcal{E} - IR - Q/C = 0$$

The equation has two unknowns - I and Q - but they are related by $I = dQ/dt$. This equation is what is known as a "differential equation" - to find Q as a function of time, we have to solve the equation. Let's take the derivative of this whole equation:

$$\frac{d}{ddt}\left(\mathcal{E} - IR - Q/C\right) = 0$$

$$-R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0$$

Battery voltage is constant, and drops out under the derivative. Thus:

$$\frac{dI}{dt} = -\frac{I}{RC}$$

We see the differential equation revealed - it's now only in terms if I and time, and we need to solve for the function of I. We can rewrite this:

$$\frac{dI}{I} = -\frac{dt}{RC}$$

If we then integrate each size - the current side from initial to final current and the time side from 0 to a time, t - we can solve for the current as a function of time:

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{1}{RC} \int_0^t dt$$

$$ln\left(\frac{I}{I_0}\right) = -\frac{t}{RC}$$

Take the exponential of each side:

$$I/I_0 = e^{-t/(RC)}$$

$$I = I_0 e^{-t/(RC)}$$

We see that the current tends to zero, starting at an initial current I_0, after a very long time, $t >> 0$. The voltage across the capacitor at any given time, t, is given by $V_C = \mathcal{E} - V_R$. Since $V_R = IR$, then:

$$V_C = \mathcal{E}(1 - e^{-t/(RC)})$$

We see from this that the product of R and C has units of *time*, and in fact RC can be identified as a characteristic time of the circuit. RC is called the "time constant," and represents the amount of time required for the voltage to rise to 63.2% of the battery voltage:

$$V_C/\mathcal{E} = (1 - e^{-1}) = 0.632$$

Discharging the capacitor

Imagine we now remove the battery and connect the capacitor through just the resistor. This now forms a simple loop circuit. Let's assume that the current now flows from the positive side of the capacitor through the resistor. This is the direction that REDUCES the current through the resistor, so $I = -dQ/dt$. Writing Kirchoff's Loop Law:

$$0 = Q/C - IR$$

We can then take the first derivative of both sides with respect to time:

$$0 = \frac{1}{C}\frac{dQ}{dt} - R\frac{dI}{dt} = -\frac{1}{C}I - R\frac{dI}{dt}$$

$$-\frac{1}{RC}dt = \frac{dI}{I}$$

$$-\frac{1}{RC}\int dt = \int \frac{dI}{I}$$

$$-\frac{t}{RC} = ln(I/I_0)$$

$$I = I_0 e^{-t/RC}$$

$$I = V_0/Re^{-t/RC}$$

$$V = V_0 e^{-t/RC}$$

Analyzing the RC circuit: short and long times

Rather than solving differential equations, you can qualitatively think about the behavior of RC circuits (short and long times after the switch it thrown, etc.).

- Just after the battery is connected to the circuit, the capacitor acts like a short in the circuit - charge is rapidly pulled from one side of the capactor, through the battery, and to the other side of the capacitor. After a little time passes, though, charge builds up on the capacitor and it begins to slow the rate of current; the capacitor offers more resistance to the flow of current.
- Long after the battery is connected, the voltage across the capacitor is opposite the voltage across the battery and current stops flowing. Now, the capacitor acts like a perfect open circuit, preventing current flow through it.

Draw an example circuit, with a capacitor and a resistor in parallel and discuss the circuit at early and late times.