General Physics - E&M (PHY 1308) Lecture Notes

Lecture 021: The Biot-Savart Law

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no tags

Goals of this lecture:

- Discuss the forces involved with magnetic fields and currents
- Introduce the *Biot-Savart* Law

Force on a single charged particle

We saw from our experiment with the Crooke's Tube that:

$$\vec{F} \propto \vec{v} \times \vec{B}$$

In fact, the exact form of this relationship is:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Review the right-hand rule.

The total force on a charged particle: electric and magnetic force

Magnetic fields influence the motion of electrically charged particles. That suggests there is some deep connection between electricity and magnetism. That said, both electric fields and magnetic fields can be present in a region of space. In that case, the total *electromagnetic force* is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

You just add the vectors to get the total force vector on a charged particle. This property is useful; you can select particles of a specific velocity if you select the electric and magnetic fields so that only particles of that velocity can survive traveling through the fields.

Units revisited

The units of magnetic field are T. A related unit is the Gauss, where $1G = 10^{-4}\mathrm{T}$. Let's figure out what Teslas are in terms of MKS units:

$$B = F/(qv\sin\theta)$$

So the units are:

$$T = \frac{\mathrm{N}}{\mathrm{C}(m/s)} = \frac{\mathrm{N}}{\mathrm{A}\cdot m} = \frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{C}\cdot m}$$

Properties of the magnetic force

The magnetic force always acts *perpendicular* to the velocity of the particle. Therefore, much like a ball swung at a constant speed around your head on a tether, the force alters the *direction* but not the *magnitude* of the velocity vector. It therefore causes a radial, but not a linear, acceleration. We can thus relate the magnetic force on a charged particle to the centripetal force. Let us define a positive force as acting OUTWARD from the center of rotation, and a negative force as acting INWARD toward the center of rotation. If we consider a magnetic field that is coming out of the board and completely perpendicular to the velocity ($\sin\theta = \sin(90^\circ) = 1$, then:

$$\vec{F}_{centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$\vec{F}_{magnetic} = -q\vec{v} \times \vec{B} = -qvB\sin\theta\,\hat{r} = -qvB\hat{r}$$

Then if:

$$\vec{F}_{centripetal} = \vec{F}_{magnetic}$$

and

$$-m\frac{v^2}{r}\hat{r} = -qvB\hat{r}$$

Considering just the strength of the two forces, we can solve for the radius of the circular motion:

$$r = \frac{mv}{qB}.$$

The greater the momentum, $p = mv$, the larger the radius (the less effect on the direction of motion of the charged particle). The greater the field or the magnitude of the charge, the shorter the radius - the stronger the effect on the motion (tighter orbit).

Magnetic force on many charges: magnetic field and current

Let's step up our discussion of magnetic field, and consider a CURRENT of electric charge and not just a single electric charge. A current is just made from a whole bunch of charge carriers, each with charge q. If we imagine that these charge carriers are moving as a speed $\vec{v}$ through a conductor that contains n charge carriers per unit volume, then we can relate the total charge to the current, carrier density, and conductor products. Recall that:

$$Q_{total} = nqAL$$

Thus the total magnetic force for a conductor of length L immersed in a magnetic field $\vec{B}$ is then:

$$\vec{F} = Q\vec{v} \times \vec{B} = nqAL\vec{v} \times \vec{B}$$

Aha! But recall that:

$$I = nqAv$$

If we write the velocity vector above as the product of a number and a unit vector:

$$\vec{v} = v\hat{v}$$

Then

$$\vec{F} = nALqv\hat{v} \times \vec{B}$$

If we DEFINE a new vector, $\vec{L}$, whose magnitude is the length of the conductor and whose direction is that of the current flow ($\hat{v}$), then we can write:

$$\vec{F} = (nAqv)(L\hat{v}) \times \vec{B} = I\vec{L} \times \vec{B}$$

Practice the concept

Draw a flexible wire passing through a region of magnetic field pointing out of the board. The wire is deflected upward. Is the current to the left or to the right in the conductor?

- ANSWER: current is to the left

Discussion

This equation gives the total force on all charge carriers in a wire whose are exposed to a uniform magnetic field. *YOu* can also apply this equation to very short segments of a wire which can be treated as being in a uniform magnetic field.

What is Magnetism?!

It was Hans Christian Oersted who is credited with setting physics and chemistry on the path to an understanding of magnetism. While magnetism has been observed since about 500-600 BC (in the Western world, it was Aristotle who gave credit for the discovery of this phenomenon to a man named Thales), it was not at all understood until the 1800s. Oersted accidentally observed that an electric current caused a compass needle to deflect. We can easily reproduce this experiment. Magnetism, therefore, seems to have something to do with the MOTION of electric charge. Not long after Oersted's publicized observation, two French scientists - Baptiste Biot and Felix Savart - performed experiements and determined the exact form of the force law for a steady current. We call this the *Biot-Savart* Law, and we'll explore it now.

The *Biot-Savart* law considers a steady current moving through a conductor. There are similarities and differences between *B-S* and Coulomb's Law. Let's write Coulomb's Law for a small piece of a distribution of charge:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

Draw a picture representing the situation that can be described by Coulomb's Law (a blob of charge, considering the electric field due to a piece of the blob).

Now draw a picture representing the situation we want to analyze in magnetic fields. We want to know the field, $\vec{B}$, at a point P some distance, r, from a part of the conductor ($d\vec{L}$) carrying a steady current I. Our convention will again be that $\hat{r}$ points from the conductor element to the point P.

Now let's think about the differences between the static charge situation considered by Coulomb's Law and the moving charge situation in *B-S* Law:

- In the BSL, we are considering a current element, $Id\vec{L}$, which is a vector quantity (it flows along a direction in the conductor). In CL, we were considering a piece of charge which had no direction. It was just a number.
- In the BSL, the source of magnetic field is a VECTOR quantity - current times $d\vec{L}$. In the CL, the source of electric field is a scalar quantity - charge. We have to account for direction of motion in the BSL.
- In the BSL, the field contribution of $Id\vec{L}$ depends on the orientiation of the conductor to the unit vector - it depends on the sine of the angle, specifically. In CL, we had no such oddity.

The *B-S* Law, which describes all of our observations, is as follows:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{L} \times \hat{r}}{r^2}$$

Here, we have a new constant that has been determined from experimental measurement: μ_0, the **permeability constant**. It's EXACT value is $4\pi \times 10^7 \mathrm{N/A^2}$. Equivalent units are often used: $\mathrm{T \cdot m/A}$.

There is one other important distinction between the BSL and CL. The CL gives us the electric field in terms of isolated charge elements. But it's impossible to talk about an isolated current element, because it necessarily must be part of a circuit. In order to get the total magnetic field, you have to integrate around the entire circuit to get the magnetic field at point P. Because magnetic field is a vector, it obeys the superposition principle so we just have to add up all the current elements:

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{Id\vec{L} \times \hat{r}}{r^2}$$

The above is the *integral form* of the BSL. The magnetic field thus depends on the details of the current distribution. Generally speaking, though, the cross-product in this law tells us that magnetic field lines encircle the path of the current perpendicular to its direction. Here we have another version of the right-hand rule:

- To determine the direction of magnetic fields around a conductor, point your thumb in the direction of current. The direction your fingers would curl around the conductor indicates the direction of magnetic field lines.