General Physics - E&M (PHY 1308) Lecture Notes

Lecture 022: Applying the Biot-Savart Law

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no tags

Goals of this Lecture

- Apply the BSL (to a line of current-carrying wire)
- Discuss the magnetic force between two conductors (parallel, infinite lines of current)
- Discuss the magnetic field of a loop of current
  - Discuss the fundamental nature of the magnetic dipole by analogy to the motion of electrons around a nucleus (classical picture)
  - Discuss the consequences of the lack of magnetic monopoles

Example: magnetic field around a straight conducting wire

Consider an infinitely long straight wire carrying a steady current I. Find the magnetic field at a point P which lies a distance y above the wire.

- Draw the wired and setup the problem. Use the new right-hand rule to note which direction (into or out of the board) we expect the field to point.

Begin by writing the BSL:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{L} \times \hat{r}}{r^2}$$

- Let's choose CARTESIAN coordinates since we have all these handy straight lines in the problem and because whatever the distance y of point P above the line, it's fixed no matter where along the conductor we are considering.

What are the unknowns?

- We need to sort out expressions for $\hat{r}$, $d\vec{L}$, and the cross-product in terms of the geometry (coordinates) of the problem
- We need an expression for r^2 in terms of geometry.

Let's attack pieces:

- r^2 is the easiest: it's just $r^2 = x^2 + y^2$
- Before trying to compute the cross-product from its pieces, let's think about it a bit:
  - Can we figure out the unit vector for $\vec{B}$? Both $d\vec{L}$ and $\hat{r}$ lie in the plane of the board. Therefore, $\vec{B}$ must point into or out of the board. From the *R-H* rule, we expect it to point OUT - always perpendicular to both $\hat{r}$ and $\vec{L}$. There - we've figured out the direction of the cross-product without multiplying a single thing.
  - We can simplify the cross product magnitude as a result of the previous observation, since we know it points out of the board. Thus $|Id\vec{L} \times \vec{B}| = IdL \sin\theta$ where θ is the angle between $d\vec{L}$ and $\hat{r}$.
  - From the geometry of the problem, we know from trigonometry that $\sin\theta = \sin(\pi - \theta) = y/r = y/\sqrt{x^2 + y^2}$.
- What about dL? Well, the way we've setup the problem, that's just dx.

We have all the pieces. Let's re-write the BSL:

$$|\vec{B}| = \frac{\mu_0}{4\pi} \int \frac{I|d\vec{L} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{dx \sin\theta}{(x^2 + y^2)} = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{y dx}{(x^2 + y^2)^{3/2}}$$

We can pull y out of the integral since we're not integrating it, and we're left with:

$$B = \frac{\mu_0 I y}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(x^2 + y^2)^{3/2}} \hat{k} = \frac{\mu_0 I}{2\pi y}$$

- What have we learned? We have learned that like the electric field from a line of static charge, the magnetic field from a line of moving charge falls of linearly with distance from the wire. But where the electric field lines point OUTWARD from the line, the magnetic field circles AROUND the line.

Magnetic attraction of two wires

What is you place two parallel lines of current next to one another, separated by a distance d? We know from the example above that the magnetic field of line 1 at the location of line 2, a distance d away, will be:

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

The field from line 1 will be perpendicular to the current in line 2, by construction, and we can just figure out the force immediately from:

$$F_{12} = I_2 L_2 B_1 = I_2 L_2 \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I_1 I_2 L_2}{2\pi d}$$

If the currents point in the same direction, the force between them is attractive. If they point in opposite directions, it's repulsive.

This force can be QUITE LARGE for large currents. Engineers have to worry about this force when designing electrical transport systems. The hum you often hear near high-voltage/high-current devices is due to the vibration from this magnetic force, as the current in such devices alternates (changes sign) and thus the force changes strength at about 60 times per second (60Hz).