# General Physics - E&M (PHY 1308) Lecture Notes

## Lecture 024: Current Loops in Magnetic Fields

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no tags

## Goals of this Lecture

- Motivate why there are no Magnetic Monopoles and write Gauss' Law for Magnetic Fields
- Explain the motion of current loops exposed to external magnetic fields

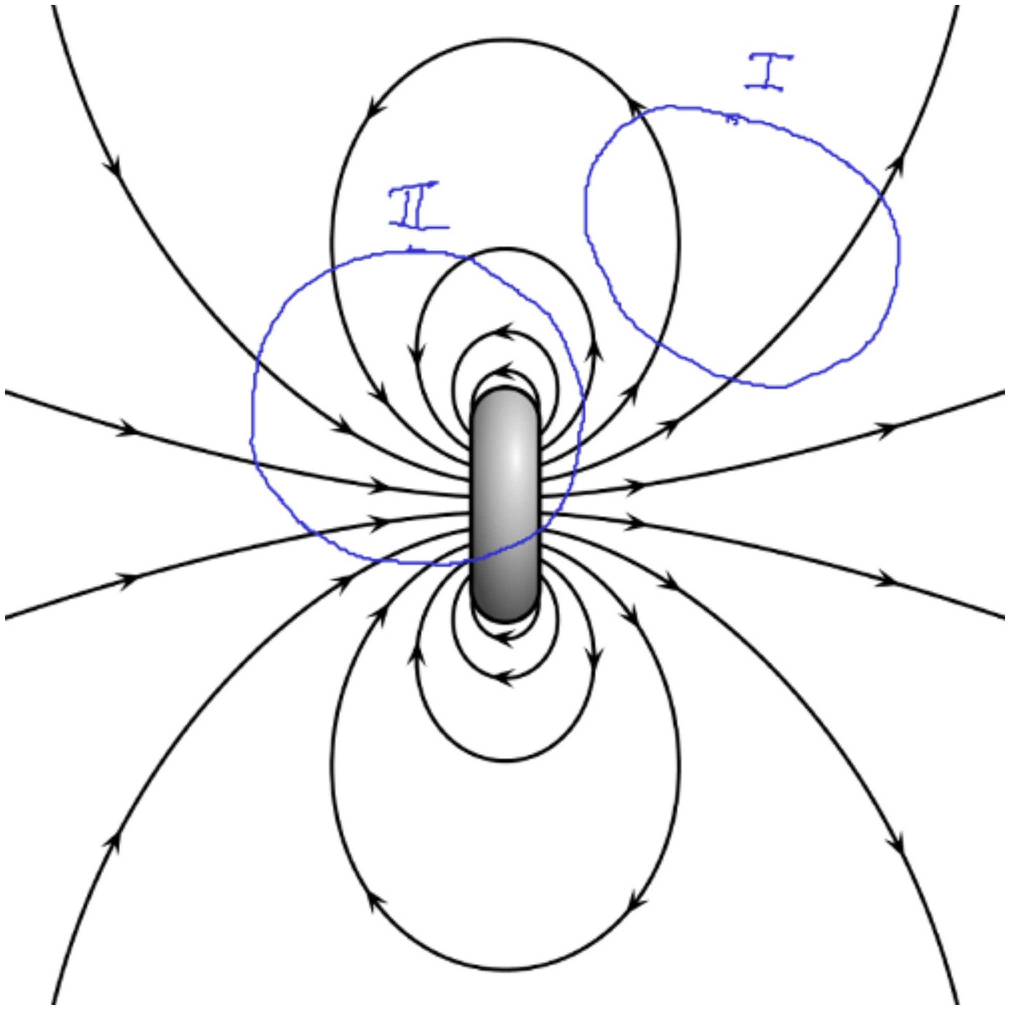
## Why there are no Magnetic Monopoles

Let's go back and consider our old friend, the electric point charge. We are able to draw a surface around a point electric charge - a Gaussian Surface - that is imaginary but which helps us to think about the flux of electric field passing into and out of the surface. If we enclosed a region near the charge that doesn't include the charge, the flux is ZERO. If we enclose the charge, we know that Gauss's Law holds, and the flux will be:

$$\Phi_E = \int_{surface} \vec{E} \cdot d\vec{A} = \rho/\epsilon_0$$

What about trying this same trick with magnetic fields? Consider our simplest field - the one due to a loop of electric current like an electron going around a proton. Let's draw surfaces see what we get.

- Draw some example surfaces. Note that no matter what we try, we always enclose a region where the net flux is zero.




- Consider surface I: In the above picture, there are 2 flux lines entering and 2 leaving. Thus the net flux, $\Phi_B$, is ZERO.
- Consider surface II: Surface II encloses part of the loop and its lines. The number of flux lines entering is 7, and the number exiting is also 7 (one of them is hidden underneath the drawing of the loop when it exits). So again, the net flux is ZERO.

If we then express the form of Gauss's Law for these results:

$$\Phi_B = \int_{surface} \vec{B} \cdot d\vec{A} = 0$$

If the right hand side of this is supposed to contain a "magnetic charge density", then what we find is that the magnetic charge density is always

ZERO, implying that there is NO MAGNETIC CHARGE.

The origin of magnetism - that is, due to the motion of electrons in atoms - leads us to the conclusion that there should be no magnetic monopoles.

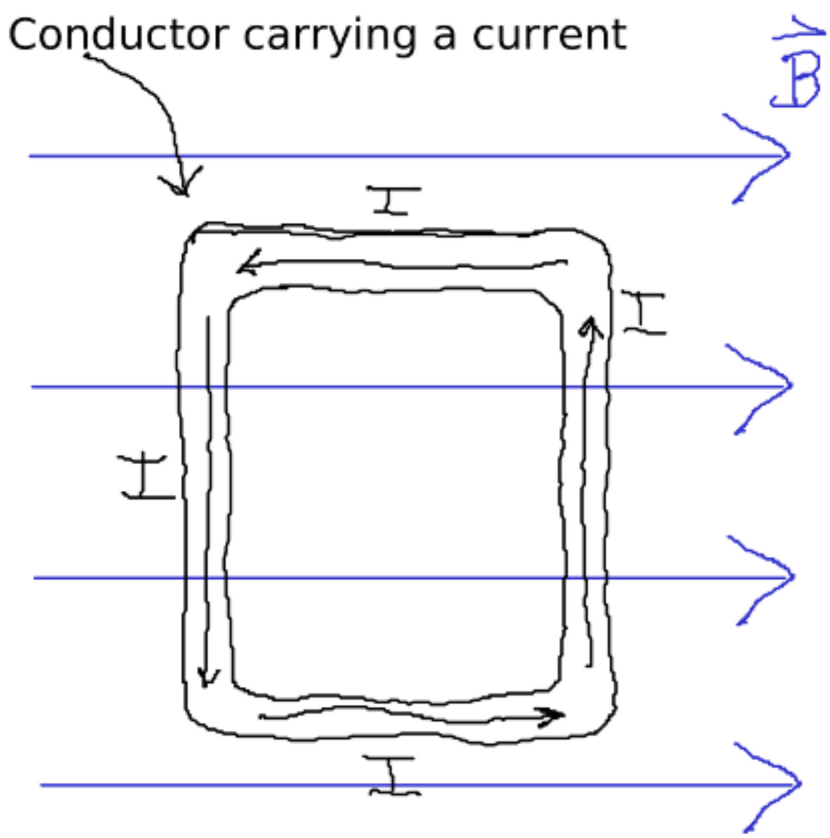
## The behavior of current loops in external magnetic fields

There is one more important phenomenon to consider: what happens when a current loop is itself exposed to a magnetic field? This is important because this problem is the basis of electric motors, which are ubiquitous in the world: from fans that cool a room, to hybrid gas-electric cars. The basis of such a motor is the force exerted on a loop of a loop of current when it is immersed in an external magnetic field.

Consider a simple geometry of a square loop. Consider also the simple case of a uniform magnetic field into which the loop can be placed. Remember that the force exerted by an external magnetic field on a current is:

$$\vec{F} = I\vec{L} \times \vec{B}$$

- What happens if we place the loop in the field so that originally the plane of the loop is perpendicular to the field?




In the drawing above, we can think about what happens by breaking the problem into 4 currents, each one moving in a different direction but all the same magnitude.

- The top current is moving from right to left, and is anti-parallel to the magnetic field. Thus its cross-product magnitude is

  $$|I_{top}\vec{L} \times \vec{B}| = ILB\sin\theta = ILB\sin\pi = 0.$$

  Therefore, there is no force on the top segment of the current.
- The bottom current leads us to the same conclusion: it is parallel to the magnetic field, and so the magnitude of its cross-product is also zero.
- Let's consider the left, downward-pointing current. Now we have a current moving at a right-angle to the magnetic field (actually, at $\theta = -\pi/2$). We then expect that the force will point OUT of the page (out of the plane of the blackboard). We can also apply the right-hand rule to see this: stick our fingers in the direction of the current, curl the fingers toward the magnetic field, and let our thumb tell us the

direction of the force - then we see that it points outward. Thus the left-hand current experiences an *outward-pointing force due to the magnetic field.*

- Finally, we consider the right-hand, upward-pointing current. The only difference between it and the left-hand current is the direction of the current. Since the direction is up, the sign of the resulting force reverses (we now have $\sin\pi/2$ instead of $\sin\pi/2$ in the cross-product magnitude), and the right-hand current *experiences a magnetic force pointing into the page/blackboard.*

So what have we learned? We have learned that a loop of current that begins fully parallel to the magnetic field will experience equal but opposite forces on the left and right side. This will cause a *rotation* - and thus there is a *torque* resulting from the force exerted by the magnetic field.

Recall that the definition of a torque is:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is a vector that points from the axis of rotation to the point where the force is exerted. In the case above, this is a vector from the vertical bisecting axis of the loop to the right or left-hand sides of the loop.

In our example above, the torque vector points vertically upward (again, use the right-hand rule for the cross-product to verify this: point your fingers in the direction of $\vec{r}$, curl your fingers toward $\vec{F}$, and your thumb indicates the direction of the torque vector.)

## More loops

What if we then wrap the wire twice, so that there are two overlapping loops of wire that start out in the plane of the paper? Now we have twice the current on each side of the loop (each wire carries current I, but there are two wires one each side now). As a result, we expect the magnetic force to double.

If we wind 3 times, we expect the force to triple. And so on. For $N$ loops, the total current is $NI$.

## The Magnetic Dipole Moment

So the magnetic force on a loop of wire is proportional to the number of windings in the loop. It is convenient to define a new vector for a current loop, just like we defined a special vector for the electric dipole that helped us to calculate the torque on a dipole in an external electric field. Recall that we defined the *electric dipole moment* as:

$$\vec{p}_{electric} = q\vec{d}$$

where $q$ is the magnitude of either charge in the dipole and $\vec{d}$ is a vector pointing from the positive to the negative charge. We found that the torque on an electric dipole could be expressed as:

$$\vec{\tau}_{electric} = \vec{p} \times \vec{E}$$

where $\vec{E}$ was the external electric field into which the dipole was placed.

Since the magnetic field of a current loop looks like a *dipole field*, we can also define the *magnetic dipole moment* of a current loop:

$$\vec{\mu} \equiv NI\vec{A}$$

The vector $\vec{A}$ will be familiar - it's just like the area vector we defined when talking about flux and Gauss's Law.

- $\vec{A}$ has a magnitude equal to the area of the loop.
- $\vec{A}$ has a direction that is perpendicular to the plane of the loop.

How do we determine the direction of $\vec{A}$? The convention is ANOTHER right-hand rule:

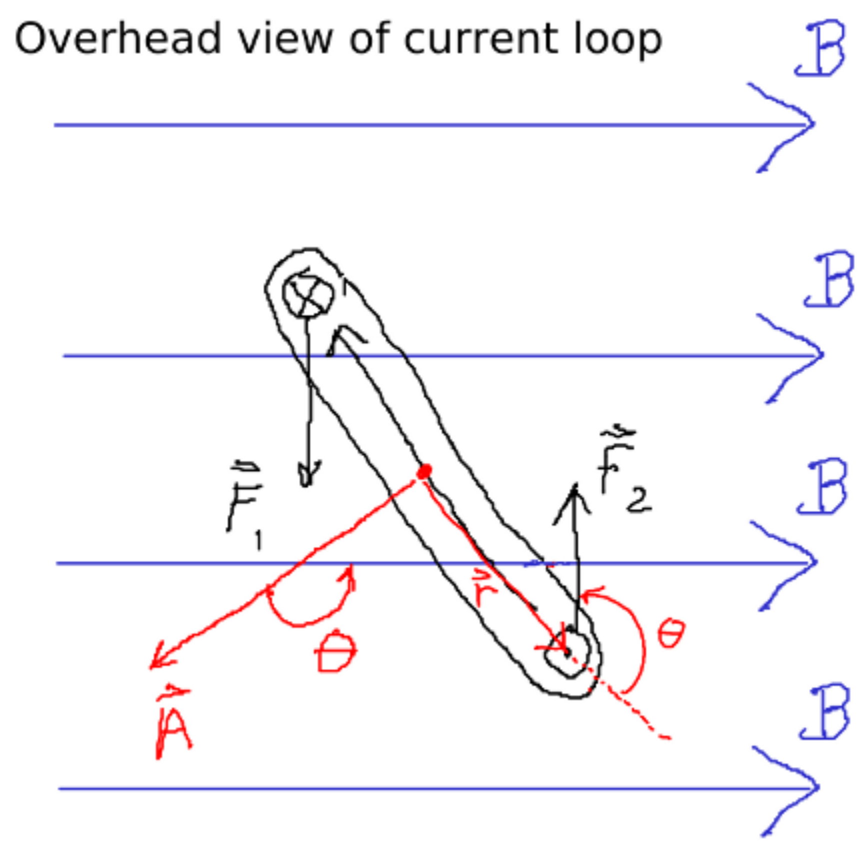
- Point your fingers in the direction of current around the loop. Your thumb points in the direction of $\vec{A}$.

(Yes, you'll have to try to memorize all of these conventions...!)

## Why Define the Magnetic Dipole Moment?

The Magnetic Dipole Moment is a convenient short-hand that let's us more quickly determine the behavior of current loops in external magnetic fields, just like the electric dipole moment let us more quickly calculate the electric force on a dipole.

Let's see why.




Consider a square current loop with N windings, like the one depicted above but with equal-length sides. The area of the loop is just:

$$A = L^2$$

The magnitude of a force on either the left or right side of the loop is always:

$$F_{side} = NILB$$

since the current on the sides of the loop is always perpendicular to the magnetic field.

The two sides are each a distance $(1/2)L$ from the rotation axis. Thus the magnitude of the torque due to the forces on **both sides** will be:

$$\tau = (\frac{1}{2}L)(2NILB\sin\theta) = NIL^2B\sin\theta$$

where $\theta$ is the angle between $\vec{r} = (1/2)L\hat{r}$ and the force exerted by the magnetic field on either side of the loop.

We can rewrite this in terms of the area, however, since the area is just $A = L^2$,

$$\tau = NIAB\sin\theta$$

which is just the magnitude of the cross-product of a vector perpendicular to the plane of the loop and at the same angle $\theta$ with respect to the magnetic field (see drawing above):

$$\vec{\tau} = (NI\vec{A}) \times \vec{B}$$

and you see why the magnetic dipole moment is defined the way that we defined it, $\vec{\mu} = NI\vec{A}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

So using just simple properties of the loop - number of windings, the current carried by the loop, its area - we can compute the rotational forces on the loop.

## The *Electro-Magnetic* Motor

The fact that a current-carrying loop will rotate in a magnetic field is the basis of electric motors (see slides). The loop is designed so that when it rotates one-half cycle, the current in the loop reverses. This causes the force from the magnets to continue to propel the loop in a circle. Correctly designed, this is a very efficient way to power something since it can be run off of electricity and has fewer moving parts in physical contact (unlike a fossil fuel motor). The challenge of this motor is energy storage: how do you store enough electricity to power it for long periods of time? This is the biggest challenge facing all-electric cars: batteries. It's why hybrids are so attractive: run a gas engine during periods of acceleration and deceleration, and run an electric engine during steady operations (which demand less power).