General Physics - E&M (PHY 1308) Lecture Notes

Lecture 025: Ampere's Law

SteveSekula, 28 October 2010 (created 28 October 2010)

no tags

Goals of this lecture

- Review integrals and introduce the line integral
- Discuss what happens when we line-integrate the magnetic field around a wire
- Motivate Ampere's Law

The Line Integral

Let's begin this lecture with a discussion of *the line integral*. Let's consider a simple path, one that closes on itself. The simplest such closed path is a circle whose radius is R. The length of the path is just the circumference of the circle: $2\pi R$.

Let's divide the path into equally sized tiny pieces, and describe each piece by vectors, $d\vec{r}$, which are vectors that each point in a direction tangent to their section of the path. The vectors describe the direction and distance you would walk if you were traveling this path in a complete loop.

The line integral is just a way of saying "add up something along the path." We denote it with a special symbol:

$$\oint$$

where the little circle reminds us that we are supposed to sum up all the little pieces around a closed path.

Let's try one of these. For instance, what if we integrate all the pieces of our circular path around the path:

$$\oint dr = 2\pi R$$

We expect this to give us the circumference - the length of the path around the circle.

The path integral of a magnetic field

Let's consider a wire carrying current I. Let's have the current point out of the page. Let's then consider a circular path around the wire, centered on the wire with radius r_1.

If we take a small piece of this path, described by a vector $d\vec{r}$, we can then take the product of the magnetic field on that piece of the path and that vector:

$$\vec{B} \cdot d\vec{r}$$

We know that the magnetic field also circulates around the wire, just like our path, and in the same direction. Thus:

$$\vec{B} \cdot d\vec{r} = Bdr$$

If we consider any piece of the path and consider this product on that piece, as long as the pieces are all at the same radius the magnetic field strength is a constant. Let's then do the path integral:

$$\oint Bdr = B \oint dr = B(2\pi r_1)$$

Recall the mathematic form for the magnetic field around an infinite wire, a radius r_1 from the wire:

$$B = \frac{\mu_0 I}{2\pi r_1}$$

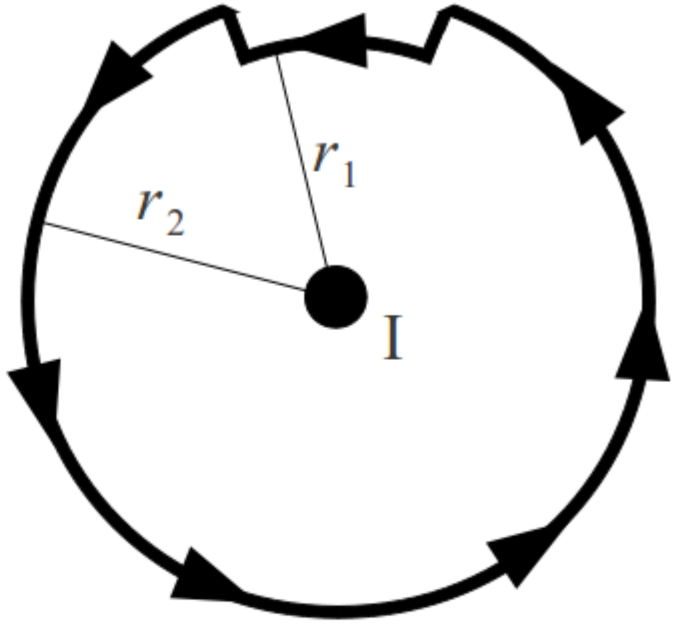
We can plug that into our line integral:

$$\oint Bdr = B(2\pi r_1) = \frac{\mu_0 I}{2\pi r_1}(2\pi r_1) = \mu_0 I$$

Huh. That's a pretty interesting result. This line integral ends up being *independent of the distance from the wire* and *dependent only on the current enclosed by the path*. Try another circular path with radius $r_2 > r_1$. You'll find the same thing:

$$\oint Bdr = B(2\pi r_2) = \frac{\mu_0 I}{2\pi r_2}(2\pi r_2) = \mu_0 I$$

In fact, any path around the wire will lead to the same result. This is seen by considering a circular path that changes radius at some point, then returns to the original radius:



We can write this path integral in four pieces, but along two of them the dot product is zero because the path is perpendicular to the magnetic field. So those little diversions don't count and we get the same result. In fact, it is universally true that:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

This is known as Ampere's Law, and we'll explore it more next time.