General Physics - E&M (PHY 1308) Lecture Notes

Lecture 027: Induction

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no tags

Goals of this Lecture

- Introduce the concept of *magnetic induction*
- Demonstrate the effect and describe it mathematically

Demonstrations

- show what a moving bar magnet does to coil of wire hooked up to a current meter ("ammeter")
- show what a pendulum made from different materials does when the materials swing through a region of high, permanent magnetic field

Magnetic Induction

- Use the *PhET* demonstrator on electromagnets (http://phet.colorado.edu/en/simulation/faraday) to illustrate the concept of changing magnetic flux

The common feature in all of these experiments is *changing magnetic flux*. If we consider the field lines from the bar magnet, we see that as we move the magnet the number of flux lines enclosed in the wire loop changes over time due to the relative motion of the loop and the magnet.

The ability for *changing magnetic flux* to induce an *electric current* is known as *electromagnetic induction*.

- We have previously been exploring the phenomenon that moving

electric charge induces magnetic fields
- We now have a new phenomenon: changing magnetic fields induce electric currents (moving electric charge)

Two scientists - Michael Faraday from England and Joseph Henry from the United States - simultaneously discovered this phenomenon. The law that we use to describe this effect is known as *Faraday's Law of Magnetic Induction*.

Magnetic Flux

The change of a magnetic field near a conductor induces an electric current, and that means there must be an accompanying induced electromotive force (emf), which normally comes from a battery but now results from changing magnetic field.

We have previously explored Gauss's Law for Magnetic Fields, which tells us that the magnetic flux through a CLOSED surface will always be zero:

$$\Phi_B(closed) = \int_{surface} \vec{B} \cdot d\vec{A} = 0$$

But what about open surfaces, like the plane of a loop of wire? In that case, it's possible to have non-zero magnetic flux. Recall the definition of flux from our discussion of electric field. Consider a vector $\vec{A}$ that is *normal* (perpendicular) to an area (e.g. the area of a loop of wire), whose magnitude A is the area of the surface. The flux is then given by:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

which is the general case when the magnetic field may depend on the location in the area where we are evaluating it. If you have a UNIFORM field and a FLAT surface, then this simplifies to:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where θ is the angle between the normal to the area and the magnetic field direction.

Example: flux through a solenoid

Consider a solenoid with N windings whose radius is R. If we run current through the solenoid, the resulting magnetic field inside the solenoid is:

$$B = \mu_0 n I$$

where $n = N/L$, the number of windings per unit length of the solenoid. The field is parallel to the axis of the solenoid and thus perpendicular to the area of the solenoid. The flux of this magnetic field through the solenoid is then:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos(0) = (\mu_0 n I)(\pi R^2) = \mu_0 \pi n I R^2$$

In this example:

- the flux through the solenoid increases with current, radius, or the number of windings per unit length - which all makes sense, because all of these things serve to either increase the strength of the magnetic field OR the area of the solenoid

Example: a non-uniform magnetic field

What is the field is NOT uniform? Then we need to apply the more general definition of magnetic flux:

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Consider a long wire in the y-direction, carrying current in the positive y-direction. Consider a rectangular circuit (loop of wire) off to the right of the current-carrying wire, whose height is L and whose width is w. The circuit has its left side a distance a from the wire, and the right side a distance $a + w$ from the wire. What is the magnetic flux through the circuit?

This is more difficult. Why? The magnetic field VARIES with distance along x from the wire:

$$B = \frac{\mu_0 I}{2\pi x}$$

That means that as we consider little patches of area on the left side of the circuit, the field is stronger than patches on the right. We need to use the integral to add all of this up. What simplifies the problem a little is that the magnetic field and area are parallel, so that the dot product is simple. The magnetic field a a given x is the same for any y, so we can divide the circuit into thin strips whose areas are each $dA = Ldx$. Then:

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int BdA = \int_a^{a+w} \frac{\mu_0 I}{2\pi x} Ldx$$

We can pull all the constants out of the integral, leaving:

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \int_a^{a+w} \frac{1}{x} dx$$

The integral just gives us the natural logarithm of x, and we can evaluate:

$$\Phi_B = \frac{\mu_0 IL}{2\pi} \left. ln(x) \right|_a^{a+w} = \frac{\mu_0 IL}{2\pi} \left(ln(a+w) - ln(a)\right) = \frac{\mu_0 IL}{2\pi} ln\left(\frac{a+w}{a}\right)$$

So we see how to tackle more complicated situations.

Faraday's Law

Michael Faraday and those that followed him discerned the exact relationship between the change in flux and the induced EMF in the circuit. Faraday's Law, along with Gauss's Laws for electric and magnetic fields, comprises three of the four fundamental laws of electromagnetism.

The law is a mathematical description of the outcome of many experiments.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Faraday's Law relates the INDUCED EMF to the CHANGE IN MAGNETIC FLUX
- The induced EMF tends to OPPOSE the change in flux, thus the presence of the minus sign. That is, the currents induced by the change in flux create their own magnetic fields which OPPOSE the change in external magnetic field.
- Flux can be changed either by changing the magnetic field strength, changing the magnitude of the area, changing the angle (orientation) between flux and area, or combinations of these.