

General Physics - E&M (PHY 1308) Lecture

Notes

Lecture 028: Faraday's Law and Energy Conservation

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Faraday's Law

Michael Faraday and those that followed him discerned the exact relationship between the change in flux and the induced EMF in the circuit. Faraday's Law, along with Gauss's Laws for electric and magnetic fields, comprises three of the four fundamental laws of electromagnetism.

The law is a mathematical description of the outcome of many experiments.

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

- Faraday's Law relates the INDUCED EMF to the CHANGE IN MAGNETIC FLUX
- The induced EMF tends to OPPOSE the change in flux, thus the presence of the minus sign. That is, the currents induced by the change in flux create their own magnetic fields which OPPOSE the change in external magnetic field.
- Flux can be changed in three ways:
 - Change the strength of the magnetic field
 - Change the area of the conductor
 - Change the orientation of the conductor and the magnetic field

(or combinations of all three)

Solving problems with Faraday's Law

- determine whether the problem has something to do with changing magnetic flux through a conductor (circuit, etc.)
- determine the expression for the flux
- differentiate the flux with respect to time, or otherwise determine the rate at which flux changes with time. This will give you the induced EMF.

Example: a changing magnetic field strength

Consider a loop of wire, radius $r = 10.0\text{cm}$ and resistance $R = 2.0\Omega$, immersed in a magnetic field whose strength is changing by 0.10T/s . The loop is arranged so that $\cos\theta = 1$ between the field and the area. Find the induced EMF.

We have to find the flux, and then determine how it changes with time. To do this, we first compute the flux:

$$\Phi_B = BA = B(2\pi r^2)$$

We then differentiate it with respect to time. The radius is constant, so the only thing that might change with time is B :

$$\frac{d\Phi_B}{dt} = 2\pi r^2 \frac{dB}{dt}$$

We've been given the rate of change of B with time, so we have everything we need to determine the induced EMF:

$$\mathcal{E} = -(3.14\text{mV})$$

The magnitude of this EMF then can tell us the current in the loop:

$$I = \mathcal{E}/R = 1.6\text{mA}$$

The meaning of the sign: Induction and Energy

Let's close this lecture with a discussion of that minus sign in Faraday's Law. What does it mean?

Think about the conservation of energy. This requires that, essentially, you get nothing for free. If you do mechanical work on a magnet to move it toward a loop of wire, and then you stop pushing, you shouldn't see the magnet continue to accelerate toward the loop free of mechanical work being done on it anymore.

Let's think about what this bedrock principle means for induction.

- Let's imagine a bar magnet on the axis of a loop of wire. The bar magnet is originally at rest. Since the magnet and loop are not in relative motion, there is no change in flux and there are no currents in the loop.
 - now consider giving the bar magnet a nudge toward the loop
 - if the currents in the loop were to appear such that the magnetic field they create ATTRACTS the bar magnet, then the bar magnet would continue to accelerate without stop even after you stop pushing. This amounts to the loop setting up a field whose "south pole" is closest to the north pole of the bar magnet. When the bar magnet is to the left of the loop, the flux is INCREASING through the loop. Once the bar magnet gets through the loop the flux would be decreasing and the sign of the current would flip, flipping the direction of the magnetic field. Now the south pole of the loop is aimed at the south pole of the bar magnet, and it REPELS the bar magnet, causing it to further accelerate
 - this would amount to something for nothing - acceleration with no cost. That violates conservation of energy.
- Reality: the current in the loop that appears when the flux starts increases OPPOSES the magnetic field of the bar magnet. In other words, the field of the current loop has a north pole facing the bar magnet north pole, repelling it and slowing DOWN the magnet. The current in the wire causes heating to occur, given by $P = I^2R$. The energy from your original push is dissipated by heating the loop, and energy is conserved.

This is called *Lens's Law*, and in short says:

"The electromotive force, and thus the current, establish by a change in magnetic flux through a conductor tends to itself establish a magnetic field that OPPOSES the change in flux."

This is the essence of the negative sign in Faraday's Law. The physics is the following:

- Faraday's law tells us that as we change the flux, the EMF is given by:

$$\frac{d\Phi_B}{dt} = -\mathcal{E}$$

- From Ohm's Law, we know that in a conductor (even a very good one): $\mathcal{E} = IR$. Thus

$$\frac{d\Phi_B}{dt} = -IR$$

- The current causes a magnetic field from the conductor that OPPOSES the change in flux.
- Energy is dissipated in the conductor, given by $P = IV = I^2R$.

This explains the conductor pendulum in the magnetic field (show the demonstration again). The conductor swings down, enters a region of magnetic field. This causes a CHANGE in magnetic flux through the conductor. Currents are established in the conductor that OPPOSE the change in flux, drawing from the kinetic energy of the pendulum. That kinetic energy, which at first causes the change in flux, is dissipated as heat in the conductor as currents at first oppose the motion, then heat the conductor.