General Physics - E&M (PHY 1308) Lecture Notes

Lecture 029: Forms of Induction and Self-Inductance

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no tags

Goals of this Lecture

- Further understand the induced current
- Define *induction* and see how it connects to circuits

Motional *Electro-Motive* Force

Faraday's Law - specifically, the negative sign - tells us that the current in a conductor moves in such a way to RESIST the change in flux.

- This is a consequence of conservation of energy. If the current moved in such a way to reinforce the change, then you would:
    - get an electric current "at no cost"
    - obtain runaway acceleration of the change in flux with no further input of energy to the system

Building some intuituion: a conductor moving in a magnetic field

We can more easily see the source of an EMF in the motion of a conductor through a magnetic field by considering a simple conductor (say, a rectangular piece of copper) that is moving through a region of uniform magnetic field pointing into the board/page. The copper is large enough so that it presents an area to the field.

In a conductor, electrons are free to move under the influence of an external force. In this case, we have electrons implanted everywhere in the conductor which then feel a magnetic force as they move to the right with the conductor.

- QUESTION: what direction is the force on the electrons? Take a vote of the class.
- ANSWER: if the conductor is moving to the right and the field points into the board, the force is DOWN due to the negative electric charge of the electrons.

So electrons drift to the bottom of the conductor, leaving a net positive charge at the top. This imbalance sets up an electric field from the top to the bottom of the conductor, and this is an EMF: a potential difference that, if the circuit were closed and flux were changing, would allow the electrons to flow as a current in the circuit.

Building more intuition: a closed circuit and changing flux

We can explore this resistance to change further using a simple model: a square current loop.

Imagine a square current loop with equal-length sides of length a. Imagine a moment when half of the loop in immersed in a magnetic field which is uniform and points into the page/board. The loop is being pulled to the right by a constant external force, $\vec{F}_{applied}$, as a velocity $\vec{v}$. We can use this simple example to understand a few things:

- Why the current in the wire moves the way that it does
- Where the energy from moving the conductor goes

A conductor contains electrons which are free from their parent atoms and can move under the influence of a potential difference, or in response to a magnetic field. When the conductor is at rest, the random motion of the electrons leads to force from the magnetic field, but that force nets to zero. However, when we PULL the conductor in an overall direction, we're giving a net direction to the movement of electrons with a net velocity, $\vec{v}$.

OK, so now we have electrons in the conductor moving in an electric field:

- QUESTION: what is the force equation for the force due to the moving

electrons in a magnetic field?
  - ANSWER: $\vec{F}_{magnetic} = q\vec{v} \times \vec{B}$

Let's think about the direction of the force on the electrons in the wire. All the electrons are moving to the right, the field points into the page, and electrons have negative charge; thus the force on the electrons points DOWN. Since the direction of current is OPPOSITE the flow of electrons, the current points UP in the wire. This gives us a clockwise current through the loop as the electrons continue to move in the conductor even once out of the magnetic field (they lose energy to collisions with atoms, and if the magnetic field goes away we expect the motion to wear down and stop once the magnetic force is gone).

Inside the area of the loop, the magnetic field of the current points INTO the page - in the same direction as the external magnetic field. As the flux of that external field decreases, we see that the current in the loop tries to reinforce the decreasing flux by adding magnetic field in the same direction.

So now we know the direction of the induced current, thanks to the magnetic force law.

- QUESTION: We have an electric current now immersed in an external magnetic field. What happens to a current in an external magnetic field?
  - ANSWER: It experiences a force due to the magnetic field, given by $\vec{F} = I\vec{L} \times \vec{B}$. The current in the top and bottom pieces of the wire move in opposite directions through equal-length conductors, so their forces are equal and opposite and cancel. The left side of the conductor feels a force that points to the left, OPPOSING the force pulling the loop to the right. Now we see what Faraday's Law implies: not only a current that acts to oppose the change in flux, but here also a force that opposes the motion (which also is a way to oppose the change in flux - reduce the force on the loop!).

Let's describe what is happening in terms of mathematics. The loop is moving to the right at speed $\vec{v}$. That means that the horizontal extent of the loop immersed in the field, given by x (where $x < a$), is decreasing at the rate:

$$dx/dt = -v$$

Let's calculate the flux in the loop. We have a uniform field and a flat area (whose dimensions are decreasing, but which is still itself flat). Thus we can use the simple flux formula:

$$\Phi_B = BA\cos\theta = BA = Bax$$

Now we can calculate the CHANGE in flux. The external magnetic field has constant strength, the vertical extent of the loop is constant, so the only thing changing is the horizontal length of the conductor in the field. Thus:

$$\frac{d\Phi_B}{dt} = BA\frac{dx}{dt} = -Bav$$

We can now compute the EMF induced in the wire by the change in flux:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -(-Bav) = Bav$$

This induced EMF drives the current in the wire, I, around the loop. We can use Ohm's Law to relate the current and the EMF through the resistance of the loop:

$$Bav = \mathcal{E} = IR$$

$$I = \frac{Bav}{R}$$

The resistance of the loop causes energy to be dissipated, and we can express that using:

$$P = I^2R = \frac{B^2a^2v^2}{R}$$

This is how much power is *disspated* by the loop, but how much MECHANICAL power was supplied by the original force? That power is given by:

What's the power being supplied by the force pulling the loop to the right? In order to maintain a constant velocity to the right, it must be true that:

$$\vec{F}_{pull} + \vec{F}_{magnetic} = 0$$

Thus the pulling force and the force on the current resisting the pull must be equal in magnitude. Then:

$$F_{pull} = IaB$$

What's the power supplied to pull the loop? The work done by the pulling force is:

$$W = \vec{F}_{pull} \cdot \vec{x}$$

where $\vec{x}$ is the distance, x, over which the force acts in a given time, t. Thus the power is:

$$P = \frac{dW}{dt} = \frac{d\vec{F}_{pull} \cdot \vec{x}}{dt}$$

If the pulling force was constant, then:

$$P = F_{pull}\frac{dx}{dt} = F_{pull}v = IaBv$$

Substituting for the current:

$$P_{supplied} = \frac{Bav}{R}(BaV) = \frac{B^2a^2v^2}{R}$$

and we see that the mechanical power put into the system to move it out of the magnetic field is EQUAL to the power dissipated by the current in the loop. Energy is conserved, which we knew had to be the case in a closed mechanical system such as this one.

Inductance

Mutual Inductance

So far we've been considering currents induced in a conductor by an external magnetic field from, for instance, a bar magnet. But as illustrated in the *PhET* demonstrator, another current creating a magnetic field (e.g. a solenoid) that then changes in the presence of a second conductor can induce a current in the second conductor. For instance, you can move the first conductor toward or away from the second, or increase the current in the first so that its field strength changes through the second conductor.

This is known a *mutual inductance*. One conductor's changing magnetic field can induce a current in a second conductor, and the second conductors change in magnetic field can induce changes in the first.

You can transmit energy using mutual inductance (ala an electric toothbrush).

Self-Inductance

There is one more critical kind of inductance. It's so critical, it's part of the basis of modern electronics. It's called *self inductance*

In reality, it's quite simple - but it's harder to think about because it's a bit like shaking one's own hand. It can seem a bit odd.

Consider a solenoid carrying a steady current. We know that this device contains a magnetic field inside the coil, and last lecture we calculated the flux inside a solenoid. What if the current were suddenly stopped?

- QUESTION: what would happen to the magnetic field inside the solenoid?
    - ANSWER: it would try to decrease to zero.

- QUESTION: what does that do to the magnetic flux in the solenoid?
    - ANSWER: decreases it

- QUESTION: what does Faraday's Law tell us will happen as a result of a decrease in flux in the solenoid?
    - ANSWER: a current will flow in the solenoid which attempts to reinforce the decreasing magnetic flux.

The ability of a flux-carrying device, like a solenoid, to induce it's own electrical current in response to a change in that flux is called *self-inductance*.

Such devices make it harder to induced sudden changes in, for instance, current in a conductor. The flip side is also that the REMOVAL of current from such a device will leave energy stored in the magnetic field, which will then induce current even after the external voltage has been removed. These currents can be DEADLY, as they can lead to a huge induced EMF from the stored energy in the magnetic field.

Inductors

A device which is designed to exhibit self-inductance and thus resist changes in current (and therefore magnetic flux) in a circuit is called an *inductor*. Inductors are critical components in modern electronics:

- They are in all radio transmitters, as they help establish and maintain the operating frequencies of transmitters and receivers.
- They appear in audio equipment and help insure that high-frequency and low-frequency signals go to the appropriate speakers (tweeters and woofers) in the system. This is how you get the right sound to the right speaker to get a crisp and authentic audio experience.

A simple example of an inductor is a solenoid. It contains many turns and the magnetic field from one side of the coil can interact with conductor on the other side of the coil, providing conditions for self-inductance.

Inductance

Consider a device placed in a circuit which has a magnetic field and that magnetic field penetrates parts of the device, yielding a potential self-inductance.

As long as the current in an inductor is steady, there is no change in magnetic flux. Thus *steady state* circuits lead to situations where the inductor makes no opposition to the circuit. In the steady state, when current flow is constant, the inductor acts like any other conducting part of a circuit.

But when the current in the circuit CHANGES, the inductor will act to oppose that change. If the current is flowing down through an inductor, and suddenly begins to decrease, the self-inductance will create an EMF in the inductor that forces current in the original direction of flow, opposing the change.

We can describe this mathematically. The flux in a device that contains a magnetic field through its own area is proportional to the current in the device. More current = more flux, less current = less flux. This proportionality is JUST LIKE Ohm's Law.

Recall that Ohm's Law related the voltage in a circuit the to current through a constant of proportionality: the resistance.

$$V = RI$$

In an inductor, flux is related to current through another constant of proportionality: the *inductance*:

$$\Phi_B = LI$$