General Physics - E&M (PHY 1308) Lecture Notes

Lecture 030: Using Inductors

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no tags

Goals of this lecture:

- Learn how inductors work in circuits to resist change
- Describe this behavior mathematically

Self-Inductance

Last time, we discussed the idea that a device that stores a magnetic field, like a solenoid, generates a field and thus establishes a flux through its own area. This device is called an inductor. Any change in that flux - for instance, the decrease or increase of current through the solenoid - will be resisted. This is called *self-inductance*.

We defined the *inductance*, L, using what should be a linear relationship between the current in the inductor and the flux in the inductor:

$$L = \frac{\Phi_B}{I}$$

What happens if the flux changes in response to a change in current through the inductor?

The behavior of an inductor

Let's imaging an inductor with a steady current passing through it. Now change the current suddenly (increase or decrease - it doesn't matter). How does the change in flux relate to the inductance of the device?

If the current changes, the flux changes in our inductor:

$$\frac{d\Phi_B}{dt} = L\frac{dI}{dt}$$

We know from Faraday's Law that if the flux changes, an EMF will be established that OPPOSES the change:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

And inserting Faraday's Law we find the EMF induced in the inductor:

$$\mathcal{E}_L = -\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$$

This is the form of Faraday's Law for an inductor with self-inductance L.

Reminder: the inductance L has units of $\mathrm{T}\cdot\mathrm{m}^2/\mathrm{A}^2$, which is given the special name "Henrys" in honor of the American Joseph Henry who co-discovered magnetic inductance. Typical inductors in electronic applications have values in the range of micro-Henrys up to several Henrys.

The EMF induced in an inductor when the current changes through the inductor is called **back emf**, and again it's an EMF that is established to resist the change in current flow. Consider the following situation:

Draw a vertical inductor symbol and an arrow pointing down representing current

- Current is flowing down through an inductor in a steady state. Suddenly the current begin to decrease. What direction is the back-EMF in the inductor (indicate by labeling the two sides of the inductor with a "plus" or "minus", like a battery, to indicate the direction of the back EMF.
- Current is flowing down through an inductor in a steady state. Suddenly the current begins to increase. What direction is the back

EMF now?

This isn't just math. Rapid changes in current through an inductor can cause currents to flow through a circuit that burn out electronic devices in the circuit. For instance, our TA (Dennis) experienced this on Homecoming Weekend when some buddies of his starting flipping on and off the solenoid valve to his confetti cannon. This fried the electronics in the device. Why? The solenoid valve, which opens and closes with the flip of a switch and allows compressed air to leave the cannon cylinder, stores energy in a magnetic field which is released when the switch is opened and closed. Done too rapidly, this will fry electronics in the device (which it did).

Inductors in Circuits

Let's write down a simple circuit containing a battery, a switch, a resistor, and an inductor. Let's consider the circuit with the switch originally open and no current anywhere in the system.

- QUESTION: If the switch is closed, what will be the current flow in the circuit just afterward?
    - ANSWER: inductors resist changes in current - that includes when the current is originally zero and then a battery tries to begin driving current through the system. Thus at very, very early times after the switch is closed we expect the current to be zero in the circuit.

- QUESTION: Long after the switch is closed, what will be the current flow in the circuit?
    - ANSWER: while the inductor will at first resist the change in current, the current should increase over time until it achieves a steady state. At this point, the current flow is no longer changing and the inductor is no longer resisting. Thus for very long times, we expect the inductor to behave like a perfect conductor, offering no resistance to current flow.

At any moment in time, we can analyze the behavior of the circuit using Kirchoff's Loop Law. Starting from just before the battery, the change in voltage through the circuit will be:

$$\mathcal{E}_0 - IR + \mathcal{E}_L = 0$$

It might at first seem strange that there is not a minus sign on the last term (the inductor EMF) - but, keep in mind that the definition of the EMF contains a minus sign,

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

We'll let that equation give us the minus sign, rather than adding it in by hand.

We can then ask, "How do quantities in such a circuit change with time?" We can answer this question by taking the time derivative of the equation:

$$\frac{d}{dt}\left(\mathcal{E}_0 - IR + \mathcal{E}_L\right) = 0$$

The battery voltage is constant, so we just have to deal with the other two terms:

$$R\frac{dI}{dt} = \frac{d\mathcal{E}_L}{dt}$$

We can then substitute for the first-derivative of the current using the inductor form of Faraday's Law:

$$\mathcal{E}_L = -L\frac{dI}{dt} \longrightarrow \frac{dI}{dt} = -\frac{\mathcal{E}_L}{L}$$

Our final equation for the behavior of the circuit is then:

$$-\frac{R}{L}\mathcal{E}_L = \frac{d\mathcal{E}_L}{dt}$$

We've encountered this equation before, when we discussed RC circuits in class. It's a first-order differential equation, because it involves both $\mathcal{E}_L$ and the first-derivative of $\mathcal{E}_L$. I'll again demonstrate how one solves a first-order differential equation of this sort (which appear in all kinds of places - cell culture population, etc.).

1. Try to collect terms involving a quantity and its differential all on one side.

$$-\frac{R}{L}dt = \frac{d\mathcal{E}_L}{\mathcal{E}_L}$$

2. Integrate both sides to try to isolate the variable.

Integrate both sides (for the range t=0,t and $\mathcal{E}_L = -\mathcal{E}_0$, $\mathcal{E}_L$):

$$-\frac{R}{L}\int_0^t dt = \int_{-\mathcal{E}_0}^{\mathcal{E}_L} \frac{d\mathcal{E}_L}{\mathcal{E}_L}$$

$$-\frac{R}{L}(t-0) = ln(\mathcal{E}_L) - ln(-\mathcal{E}_0)$$

$$-\frac{R}{L}t - ln\left(\frac{\mathcal{E}_L}{-\mathcal{E}_0}\right)$$

3. This involved a natural log. We can "undo" that using the Euler Number.

Raise both sides of the equation into the exponent of the Euler Number. Thus "undoes" the natural logarithm on the right side.

$$e^{-(R/L)t} = \frac{\mathcal{E}_L}{-\mathcal{E}_0}$$

Finally:

$$\mathcal{E}_L = -\mathcal{E}_0 e^{-(R/L)t}$$

where again the minus sign indicates that this EMF in the inductor OPPOSES the battery EMF from the moment the switch is closed.

We see that we have exponential behavior in the circuit. The "time constant" of the circuit is given by $\tau = L/R$, has units of seconds, and describes the amount of time needed for the EMF in the inductor to achieve:

$$\mathcal{E}_L = -\mathcal{E}_0(1/e) = -\mathcal{E}_0(0.368)$$

It's a number that tells us the "typical time behavior" of the system.

The current in the resistor will be given by:

$$IR = \mathcal{E}_0 + \mathcal{E}_L$$

(by rewriting the loop law). Then:

$$I = \frac{\mathcal{E}_0 + \mathcal{E}_L}{R}$$

And plugging in our solution for the inductor emf:

$$I = (\mathcal{E}_0/R)\left(1 - e^{-(R/L)t}\right)$$

So we see that at t=0 (the switch is closed), the current in the resistor is zero ($e^0 = 1$). As time gets very big (approaches infinity), the current approaches the limit $I = \mathcal{E}_0/R$, which is the current through the resistor if the inductor were replaced with ideal conductor.

Analyze a circuit

Consider a circuit with a resistor in parallel with an inductor. These are in series with another resistor, and then a switch and a battery.

- The switch is initially open.
- QUESTION: what does the current in the circuit look like just after the switch is closed?
- QUESTION: what does the current look like after a very long time?
- QUESTION: what does the current look like if you then re-open the switch after a long time has passes?