# General Physics - E&M (PHY 1308) Lecture Notes

## Lecture 031: Energy in the Magnetic Field

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no tags

### Goals of this Lecture:

- Understand energy stored in the magnetic field

### The inductor as a teaching tool

The inductor can teach us many things, the last of which is how energy is stored in a magnetic field. Recall from last time we described a circuit containing a battery, a switch, a resistor, and an inductor in series. We wrote down the Kirchoff's Loop Law equation for the circuit and found:

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

### Magnetic Energy

We want to understand how energy changes in the circuit. To do this, we can multiply the whole equation by the current in the circuit and obtain expressions for the power dissipated through the resistor:

$$I\mathcal{E}_0 - I^2R - LI\frac{dI}{dt} = 0$$

The first term is the power delivered to the circuit by the battery. The second is the power dissipated through the resistor. The third term is interesting:

$$P_L = LI\frac{dI}{dt}$$

This is the power stored by the inductor.

- We can see this by considering the case where current is INCREASING in the circuit. In that case, $dI/dt > 0$ and the inductor is sapping power from the circuit - storing it in the magnetic field.

- If we instead decrease the current after it achieves steady state, $I > 0$ and I = constant, then we see that the sign of this terms becomes overall positive and the inductor DELIVERS power to the circuit.

The total energy stored in the inductor can be determined by integrating the power with respect to time:

$$P_L = \frac{dU_L}{dt}$$

$$U_L = \int P_L dt = \int LI\frac{dI}{dt}dt = \int LIdI = \int_0^I LIdI = \frac{1}{2}LI^2$$

Thus the total energy stored in the magnetic field of an inductor is given by:

$$U_L = \frac{1}{2}LI^2$$

Recall that the energy stored in a capacitor's electric field is:

$$U_C = \frac{1}{2} C V^2$$

There is a LOT of symmetry between how capacitors behave in circuits, storing energy in the electric field, and how inductors behave in circuits, storing energy in the magnetic field.

## Total magnetic energy in an inductor

What is magnetic energy stored PER UNIT VOLUME in an inductor? This is the *magnetic energy density*. We can determine this for a solenoid of length $\ell$ and area A, for example.

Let's compute the energy stored in this example inductor.

- We need the flux of the solenoid. The total flux is the flux per turn multiplied by all the turns in the solenoid, or:

$$\Phi_B = NBA = N(\mu_0 n I)A = (n\ell)(\mu_0 n I)A = \mu_0 n^2 I A \ell.$$

- We then can compute the inductance of the solenoid:

$$L = \frac{\Phi_B}{I} = \mu_0 n^2 A \ell.$$

- Finally, we can insert this into the magnetic energy equation for the inductor and try to simplify this as much as possible:

$$U_L = \frac{1}{2} \mu_0 n^2 I^2 A \ell = \frac{1}{2\mu_0} (\mu_0 n I)^2 A \ell = \frac{B^2}{2\mu_0} (A\ell).$$

What's interesting is that the last piece of this equation is $(A\ell)$, the VOLUME of the solenoid. We can make this equation generic to ANY magnetic field by computing the energy density in the magnetic field - the energy divided by the volume of the solenoid:

*Missing close brace*

Does look familiar? Recall that the energy density of the electric field:

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

## The symmetry of electric and magnetic fields

This weird symmetry in the math of electric and magnetic fields is not an accident. We'll explore this in Monday's lecture, in which I will show you how what we have so far learned about electric and magnetic fields can explain the nature of light. But for now, simply note the following curiosity:

What is the total energy in a region of space that contains both an electric and magnetic field? If that space has a volume, V, then the total energy density will be:

$$u_{total} = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0(E^2 + \frac{1}{\mu_0\epsilon_0}B^2)$$

Here's the curiosity, something we will explore a bit more on Monday: what is that number that now multiplies $B^2$?

$$\frac{1}{\mu_0\epsilon_0} = \frac{1}{(4\pi \times 10^{-7}\mathrm{N/A^2})(8.85 \times 10^{-12}\mathrm{C^2}/(\mathrm{N} \cdot m^2))} = (8.99 \times 10^{16})\mathrm{m^2/s^2}$$

Hmm. Weird. That thing has units of speed-squared. Shall we take the square root and see what speed this is?

$$\frac{1}{\sqrt{\mu_0\epsilon_0}} = (2.998 \times 10^8)\mathrm{m/s}$$

Well, holy crap. That's the speed of light.

We'll find out why the speed of light plays a central role in the energy-density of electric and magnetic fields in the next lecture.