General Physics - E&M (PHY 1308) Lecture Notes

Lecture 037: Applications of Lenses - Corrective Optics and the Telescope

SteveSekula, 3 December 2010 (created 3 December 2010)

no tags

Application of Lenses: The Human Eye

The human eye is a complex optical system that can be treated using the principles we have so far developed:

- reflection and refraction
- lens properties

The biological eye can do many things to alter its optical properties - it is by no means a "fixed lens" system:

- it can vary the focal length of its optics
- it can vary the amount of light entering the optical system

Let's discuss the eye and its individual optical components:

- the cornea: this is a thin layer of material that is the first dense medium that light encounters. It possesses a curved surface, which is fixed in shape. Most of the refractive focusing occurs in the cornea, which is why its shape is so critical to the proper functioning of the eye
- light then encounters the aqueous humor, a liquid material between the cornea and the lens. One of the functions of this liquid is to maintain pressure on the cornea and help to keep its curved shape.
- light then encounters the iris. This is a muscle controlled opening that can dilate or constrict depending on whether the eye needs to allow in more or less light (in order to adjust to darker or brighter external

conditions).

- light then passes through the \m^{-1} lens is a flexible structure whose shape can be adjusted by muscles in the eye. The shape of the lens - the radii of curvature on either side - determines the focal length (as given by the lensmaker equation). Thus to change the focus of the eye to near or far objects, your eye needs only to compress or relax the lens.
- The vitreous humor: this liquid lies behind the lens and fills the eye ball
- The retina: this is a lining of two kinds of cells - rods and cones - which convert light of different frequencies into electrochemical signals. These signals travel along the optic nerve to the brain, where they are interpreted as what we call "sight".

Some features of the eye:

- Even though it has VERY good control of the optics, the eye is not really able to focus on objects that are closer than 25cm. This is called the *near point*, and corresponds to the object distance s at which you can place an object and still focus it on the retina.
  - I measured this on my own eye last night by putting one end of a rule to the right of my right eye socket and pointing the ruler straight out from my face. I then brought a piece of paper closer to my eye until I started to lose focus. With my glassed on, this occurred at 22cm.
  - the near point gets further away as we age, due to the hardening of the lens material in our eye.
  - A normal eye, without the use of corrective optics, is essentially a simple object, lens, real image system
    - question: is the real image formed on the retina upright or inverted?
      - ANSWER: Inverted. Our brain is accustomed to correcting for this effect, so we never really notice it.
  - The vertebrate eye has an annoying flaw - we have a blind spot. That's because as the vertebrate eye evolved, the retina evolved to readout the cells from the front, not the back, of the retina. Where the individual nerves cluster into the optic nerve, we have no cells to receive light and thus a blind spot in our vision. Cephalopod eyes, which are superficially similar to vertebrate eyes, DO NOT have this flaw. Instead, their retina is read out from BEHIND the cells, preventing the blind spot where the nerved bundle into the optic nerve.

        - It was French Physicist and Priest Edme Marriotte who first identified the blind spot in 1660. In doing so, he disproved a hypothesis in circulation at the time that the location where the optic nerve bundled was the most SENSITIVE part of the eye.

Disorders of the eye:

- Common optical disorders are as follows:
    - Nearsightedness:
        - A nearsighted eye can see close objects but not those that are further away. The nearly parallel rays of distant objects enter the eye and are over-focused by the eye - ahead of the retina. Distant objects appear blurry. Corrective optics are needed for this, specifically a lense that diverges the parallel rays so that the over-strong eye will focus the light further back in the eye, on the retina. The diverging lens - for instance, glasses or contact lenses - must create a virtual image for the eye that is closer than the object. Since the eye can focus clearly on nearby objects, the lens must make it seem that the object is closer than it really is. This means that we have a positive object distance, s, but a negative image distance, $s' < 0$, so that light from this image can be focused on the back of the eye.
    - Farsightedness:
        - Farsightedness is just the opposite. A Farsighted eye can see distant objects, but has a hard time seeing close objects (like text). In this case, the eye's focusing power is too weak to make the larger-angle rays from close objects converge on the retina. You need to help the eye by pre-focusing the light - you need a *converging* lens. The corrective lens again creates a virtual image, this time FURTHER from the eye than the object. The

unaided eye can see more distant things, so this works with the eye to focus nearby objects on the retina. Again, the image distance for the corrective optics is negative.

Corrective lenses

Corrective lenses are VERY common. Most of us will need them at some point in our life (for instance, as the lens ages and hardens, it becomes hard to focus on nearby objects - you get more farsighted. You need converging lenses - reading glasses - to help).

Optometrists speak of the "power of corrective lenses" - this power is called *diopters*, and is simply the inverse of the focal length of the corrective lens.

- 1 Diopter corresponds to a focal length of $1\mathrm{m}$ - thus 1 diopter = $1/(1\mathrm{m})$.
- 2 Diopters corresponds to a focal length of $0.5\mathrm{m}$ - thus 2 diopters = $1/(0.5\mathrm{m})$.

Diopters are just m^{-1}.

To correct your vision, you want to adjust the focal length of the eye using lenses that changes the near-point from whatever your eye sees unaided, to the standard near point.

For example, if you go on vacation and forget your reading glasses (e.g. you're farsighted), you can do a simple test to determine what reading glasses to buy in the store. Figure out how far away you need to hold a book in order to focus on it. This is the distance where you want the corrective lens to place the virtual image so that you can see objects that are actually closer - at the near point of $25\mathrm{cm}$. So if you find that you have to hold the book $70\mathrm{cm}$ from your eye, you know that whatever reading glasses you buy they have to have lenses that place virtual images of the text at $-70\mathrm{cm}$ in front of your eye.

Now, you want to correct the near point to $25\mathrm{cm}$. Thus, the lens equation for the focus of the corrective lens becomes:

$$\frac{1}{s} + \frac{1}{s'} = 1/f$$

$$\frac{1}{0.25\mathrm{m}} - \frac{1}{0.70\mathrm{m}} = 2.57\mathrm{m}^{-1} = 2.57\,\mathrm{diopters}$$

Thus you want to buy reading glasses that correct with a power as close to 2.57 diopters as possible

Question: why is it that contact lenses can be so thin?

The answer comes from the lensmaker equation: curvature is focus, and focus is corrective power. It's all about the radii of curvature of the contact lens, NOT its thickness.

Application of Lenses: the Refracting Telescope

It was the perfection of the refracting telescope as an instrument of defense (for viewing distant ships approaching Italy) that catapaulted Galileo Galilei into fame. The academic freedom afforded by the success of his telescope allowed him to pursue pure science. He turned the telescope on the heavens and saw things no human had ever seen before: mountains on the moon; moons circling Jupiter; spots on the sun.

The telescope he used is called a *refracting telescope* and consists of two converging lenses. The first one focuses parallel light at its focus. The image formed by the first lens - the objective lens - is within the focal length of a second converging lens called *the eyepiece*. This creates a second image that a viewer on the other side of the eyepiece sees as coming from the same side as the first image. Thus a refracting telescope creates an *enlarged virtual* image of distant objects.

The angular magnifying power of a telescope is given by the ratio of the two focal lengths:

$$m = \frac{f_O}{f_E}$$

To magnify Jupiter to the point where you could clearly see its largest moons, Galileo only needed a telescope that could magnify by a factor of 30. His telescope was fairly compact, less than a meter long. If we assume the focal length of the objective lens was about 0.5m, then the focal length of the eyepiece needs to be:

$$30 = \frac{0.5\mathrm{m}}{f_E}$$

$$f_E = 0.5\mathrm{m}/30 = 0.017\mathrm{m} = 1.7\mathrm{cm}$$

These are fairly modest designs. Galileo's lens-making technique was more than sufficient to design such simple lenses, and it's clear why he was able to succeed. His genius was in using the telescope for something nobody else had done before, and having the mathematical ability to translate what he saw into math.

Lecture 036: Application of Lenses - the Human Eye

SteveSekula, 3 December 2010 (created 30 November 2010)

no tags

Goals of this lecture

- conclude the discussion of thick and thin lens properties
- discuss the human eye

Snell's Law for a Thick Curved Refractor

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

The incident angle α appears nowhere in this equation. Thus this equation holds for ANY small angle α. Light comes to a common focal point inside the lens.

This equation is not just useful for real images:

- It applies to virtual images, $s' < 0$
- It applies to concave surfaces is we take $R < 0$
- It even works for a FLAT surface, if we take $R = \infty$. Thus it can be applied to thick plate glass windows, for instance.

Lenses: thick and thin

We can now apply this more generic version of the lens equation to thick and thin lenses. Let's consider a thick lens, surrounded by air $n_1 = 1$, with two different curvature radii on the two sides of the lens.

We can treat thick lenses in two stages:

1. Light enters the lens from one side from the object at position O. The light refracts on the surface with curvature R_1 and forms an image I_1. We can use the above equation to find that image location
2. Light from image I_1 then exits the other side of the lens, with curvature R_2. It refracts at that surface and forms a second image, I_2

Treating the two halves of the problem, for the left side (image 1):

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1}$$

Image 1 becomes object 2, and $s_2 = t - s_1'$ (where t is the thickness of the lens). Applying the lens equation again:

$$\frac{n}{t - s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2}$$

The Thin Lens Approximation

If we then make this lens thin, such that $t << R_1$ and $t << R_2$, this means essentially sending the thickness of the lens to zero. We then have:

$$\frac{1}{s_1} + \frac{n}{s_1'} = \frac{n-1}{R_1} \text{ (left side)}$$

$$-\frac{n}{s_1'} + \frac{1}{s_2'} = \frac{1-n}{R_2} \text{ (right side)}$$

Adding the two equations, we obtain:

$$\frac{1}{s_1} + \frac{1}{s_2'} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)(n-1)$$

The left-hand side here is equivalent to the left-hand side, and we can drop the primes and subscripts because the equation only contain the object distance and the FINAL image distance. Thus the right-hand sides must be equivalent:

$$1/f = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)(n-1)$$

This is the *lensmaker's formula* and tells you how to make a lens with a desired focal length from a material n and the appropriate grinding of the two sides of the material to make curved surfaces with radii R_1 and R_2. The radii can be positive or negative; they are positive when a convex surface faces the incident light, and negative when a concave surface faces the incident light.

DIfferent kinds of lenses

We can use the lensmaker equation and think about what it means to make different kinds of lenses. For instance, if we want to make a converging lens we want to make a lens that focused light from the object into a real image on the other side of the lens. Thus we need a POSITIVE focal length. We can achieve that if the radius of curvature of the left side is positive, giving us a convex shape, and the radius of curvature of the right sight is negative, giving us a concave shape.

We can make a diverging lens - one whose focal length is on the left side of the lens where the object is located - by instead doing one of two things:

- making a double-convex lens where the right radius of curvature is smaller than the left
- making a concave-convex lens where the left radius of curvature is negative and the right radius of curvature is positive.

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