General Physics - E&M (PHY 1308) Lecture Notes

Lecture 003: Electric Field and Simple Distributions of Charge

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no tags

Goals of this lecture

- Discuss in detail Coulomb's Law
  - two point charges
- Discuss principle of superposition
  - more than 2 point charges
- Discuss the concept of the electric field, the FOUNDATIONAL concept of electricity and magnetism
  - allows us to not worry about what charge is being acted upon and thus envision complex geometries
- Discuss the electric dipole
  - dipole force and field, torque, etc.
- Discuss how to handle very large distributions of charge
  - calculus!

Electric Force

As indicated by demonstrations from last lecture, electric charge is able to exert a force. We tend not to notice this force most of the time because the electrons and protons in our bodies, and in the work around us, are largely paired up and thus electrically neutral (zero electric charge) on a human scale.

As the soda can and the candle flame demonstrations show, electric charge and force go hand-in-hand at both the microscopic (ions in the candle flame) and macroscopic (soda can attracted by charged plastic) levels. Many observations and measurements of the relationship between:

- The magnitude of the charges involved
- the distance between the charges (it's direction AND magnitude)
- the sign of the charges
- the force exerted between charges

have been carefully studied. The result is mathematical statement that has been upheld by thousands upon thousands of repeated experiments carried out over hundreds of years - a LAW. This law is known as "Coulomb's Law":

$$\vec{F_{12}} = \frac{k \cdot q_1 \cdot q_2}{r^2}\hat{r}$$

where $\vec{F_{12}}$ is the force VECTOR (magnitude and direction) that charge 1 exerts on charge 2. k is a constant, determined from repeated experimentation, whose value is:

$$k = 9.0 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$$

Let's draw a picture of this and illustrate the pieces of this formula. It combines two key areas of mathematics: standard algebra and vector algebra. The picture will help us to parse the meaning of this formula, considering two cases: a pair of like-signed charged, and a pair of opposite-signed charges.

When solving a problem it's good to develop a strategy for attacking thinks step-by-step with Coulomb's Law.

Strategies for dealing with problems involving the force between two particles

Remember: Coulomb's Law reminds us that force is a VECTOR, and thus Coulomb's Law provides us both with the MAGNITUDE and the DIRECTION of the force exerted by charge 1 on charge 2

1. Interpret the problem to determine what you need to figure out: First, make sure that you're dealing with the electric force alone, and that no

other forces also need to be considered. Identify the charge or charges on which you want to calculate the force. Next, identify the charge or charges producing the force - the SOURCE charges. In the above example, we implicitly chose charge 1 and the source and wanted to know the force it exerted on charge 2.

2. Develop a set of information for attacking the problem: Begin with a DRAWING that shows the charges. If you're given coordinates for the charges, place the charges on coordinates; if not, define a suitable coordinate system that makes the problem easier. Determine the unit vectors in Coulomb's Law ($\hat{r}$). If two charges lie along the same coordinate axis, then the unit vector will be one of $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$. When the charges don't lie simply on a single coordinate axis, find the unit vector by writing the vector $\vec{r}_{12}$ and determining the unit vector $\hat{r}$ as follows:

   $$\hat{r} \equiv \frac{\vec{r}_{12}}{|\vec{r}_{12}|}.$$

   Please note that for simplicity, we will in the future write

   $$|\vec{r}_{12}| = r_{12}.$$

3. Evaluate the electric force: Apply Coulomb's Law, using the pieces you assembled during the Interpretation phase of the problem

1. Assess the outcome: Think about your answer to see if it makes sense. Check the little things: does you force point in the direction you expect, based on the charges involved? Compare to an order-of-magnitude estimate of the answer: ignoring the specific values of the numbers involved and multiple just by the appropriate powers of ten, does your answer get within a factor of about 10 of your fully computed answer?

Problem Solving: Coulomb's Law

To attack a problem involving Coulomb's Law, you need to keep a few definitions in mind:

- $\vec{F}_{12}$ is the *force that charge 1 exerts on charge 2*
- q_1 is the charge of the source charge (and is a signed quantity) and q_2

is the charge of the target charge (the one on which you are trying to determine the force)

- the unit vector $\hat{r}$ always points from the *source charge* to the *target charge*
- double-check any results using what you know about charges:
  - like charges REPEL
  - unlike charges ATTRACT

See lecture slides for a demonstration calculation

Point Charges and the Principle of Superposition

When dealing with more than one pair of charges, you need a strategy for computing the force on a charge, Q, given a number of other charges q_i (where i runs from 1 to N and labels each of the remaining charges). Because force is a vector, to find the total force on Q you add the forces (vectors) exerted on Q by the charges q_i. The force that q_1 exerts on Q is unaffected by the force q_2 exerts on Q - this allows us to superpose the individual forces to find the total force. This is not obvious, but its reality has been upheld by experiments and observations of nature. Nature didn't have to be this simple, but it is.

Why do you need this? Coulomb's Law applies to **point charges** - charged objects whose size is negligible. However, the real world is populated by **charge distributions** - a collection of many charges spread out over space. For instance:

- molecules are an example of distributions of charges - protons and electrons - and those distributions matter when you are thinking about how different molecules interact with one another (and, since they are similarly sized, you cannot neglect their dimensions).
- your heart contains a charge distribution, which accumulates during systole (contraction of the heart) and causes heart muscle tissue to contract and pump blood

Therefore, we are often confronted with situations where we need to deal with a distribution of charge.

Review of the Field Concept

Forces like gravity bothered scientists in the 1600s because you had to invoke "spooky action at a distance" to explain how, for instance, the earth kept the moon in orbit. The idea of a field relieves the mind of the concern about a mysterious and unseen contact between two objects; instead, it introduces the idea that, for instance, the earth creates a gravitational field and the moon responds to that field.

In gravitation, we talk about the acceleration due to gravity. That can be written:

$$\vec{g} = \vec{F}/m.$$

The gravitational acceleration can then be thought of as the force per unit mass that an object in Earth's gravitational field would experience. $\vec{g}$ becomes the *gravitational field*, and it is defined as the force per unit mass at any point in space around the mass.

Electric Field

We define the electric field similar to the way that you have learned about the gravitational field. It was Michael Faraday (1792-1867) [http://en.wikipedia.org/wiki/Michael_Faraday] who introduced the idea of an electric force field. Again, he did so to explain the "spooky action at a distance" that objects appeared to experience in the presence of electric charge.

Demonstration: the Van de Graaff Generator

If you've never felt an electric field before, after this you'll believe they exist.

Describing the electric field

The electric field is given by:

$$\vec{E} = \vec{F}/q,$$

the *force per unit charge* experienced by a charged object at any point in space. The electric field exists everywhere in space, and we represent that field by a series of vectors showing the force experienced by a charge q at the corresponding points in space.

Explore the electric field concept through visualization:

- http://phet.colorado.edu/en/simulation/efield

The idea of a field can be quite abstract, at first, but it's a useful idea that pervades physics (in fact, "Quantum Field Theory" is the underlying mathematical description that we have of nature). In the laboratory, you can map out the electric force field by measuring the electric force over a large number of points around a point charge, or a series of charges (e.g. between two sheets of charge).

We have to be a little careful with the field concept. The q in the field equation above is assumed to be small relative to the charge, or distribution of charge, that we are probing. This is so that the field of the charge itself can be neglected. This "test charge" idea is useful for visualization, but be careful with it.

Connecting Coulomb's Law of Force to the Electric Field concept

We have a concept of the electric field. Let's connect it to Coloumb's Law and determine the mathematical formula for the **electric field of a single point charge**:

$$\vec{E} = \vec{F}/q = \left(\frac{kQq}{r^2}\hat{r}\right)\frac{1}{q}$$

The units of electric field are N/C (Newtons per Coulomb).

- Fields of hundreds to thousands of N/C are commonplace
- Fields of 3 MN/C (3×10^6 N/C) will tear the electrons off of air molecules

The above equation for the electric field is so closely related to Coulomb's Law that it is often referred to simply as "Coulomb's Law." Since $\hat{r}$ always points away from Q, the electric field extends OUTWARD from positive charge and INWARD to negative charge.

The Electric Fields of Charge Distributions

More commonplace in nature than a single charge are *distributions of charge*. Just as with the aggregate force from a set of charges, the combined electric field from a distribution of charge is obtained by the vector sum of their individual electric fields:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... = \sum_{i=1}^{N} \vec{E}_i = \sum_{i=1}^{N} \frac{kq_i}{r_i^2} \hat{r}_i.$$

The Electric Dipole

An electric dipole is simply an object consisting of **two point charges of equal magnitude by opposite sign**. The ends are joined in some rigid mechanical way (e.g. through a physical expanse of material, or a chemical bond).

Examples of electric dipoles:

- water molecules
- Radio and TV antennas
- The heart (during systole, or the contraction phase).
    - an electrocardiogram is a measure of the strength of the heart dipole that forms when it contracts (which happens as cells move ions around to create a charge imbalance)