General Physics - E&M (PHY 1308) Lecture Notes

Lecture 004: Electric Fields and Their Effect on Matter

SteveSekula, 29 January 2011 (created 26 January 2011)

no tags

Goals of the Lecture

- Discuss different kinds of electric field
  - Uniform and non-uniform
- Discuss the details of the dipole electric field
  - What does a dipole do when exposed to an electric field (use uniform field as an example)

Comments

- What do I do when a vector's tail is not at (0,0)?
  - Let's say the tail has coordinates (2,0) and the head of the vector has coordinates (5,4). To get the vector from the origin to the head, you do this:

$$\vec{v} = (5,4) - (2,0) = (3,4) = 3\hat{i} + 4\hat{j}$$

Different kinds of electric fields

Last time we discussed the *electric field of a point charge*. The general definition of electric field is *force per unit charge*:

$$\vec{E} = \vec{F}/q$$

Using Coulomb's Law for the force field around a point charge, we get the electric field of a point charge:

$$\vec{E}_{point} = k\frac{Q}{r^2}\hat{r}$$

This is an example of a *non-uniform electric field*. How can you tell? If you draw the electric field lines, and consider the number of field lines passing through a fixed area, that changes depending on where we draw the area. A *non-uniform* electric field exerts different strength forces at different locations. The electric field is a function of place, to put this in mathematical terms.

A *uniform electric field*, by comparison, has the quality that the strength is constant no matter where you are. Draw the same area in different locations, and you count the same number of lines entering the area no matter where you put the area. A uniform electric field has the property that the strength is NOT a function of position. Its a constant, independent of place. The force exerted by a uniform field is then:

$$\vec{F}_{uniform} = q\vec{E}$$

A uniform field is useful. If you can approximate an electric field as uniform and attack a problem that way, you can ignore position-dependent effects.

The electric field of a dipole

An electric dipole has more structure than a single, lonely charge:

- Its net charge is ZERO
- It still possesses an electric field, owing to the *slight separation* of the two charges. Far enough away, the dipole separation becomes so small that it is negligible, and it should appear electrically neutral.

Let's consider the example of a dipole, and determine the electric field of a dipole and how that field changes with distance. This will be a class exercise. Half the class in their teams will work the electric field from the left charge in a dipole, and the other half will calculate the electric field from the right charge. See slides for the setup.

Pieces:

- On the perpendicular bisector:
  - Electric field from negative charge (left):

$$\vec{E}_{-} = k\frac{-q}{(y^2+a^2)}\frac{(a\hat{i}+y\hat{j})}{\sqrt{y^2+a^2}} = -\frac{kq(a\hat{i}+y\hat{j})}{(y^2+a^2)^{3/2}}$$

  - Electric field from positive charge (right):

$$\vec{E}_{+} = k\frac{q}{(y^2+a^2)}\frac{(-a\hat{i}+y\hat{j})}{\sqrt{y^2+a^2}} = -\frac{kq(a\hat{i}-y\hat{j})}{(y^2+a^2)^{3/2}}$$

- On the axis of the dipole:
  - Electric field from the negative charge (left):

$$\vec{E}_{-} = k\frac{-q}{(x+a)^2}\frac{(x+a)\hat{i}}{(x+a)} = -\frac{kq\hat{i}}{(x+a)^2}$$

  - Electric field from the positive charge (right):

$$\vec{E}_{+} = k\frac{q}{(x-a)^2}\frac{(x-a)\hat{i}}{(x-a)} = \frac{kq\hat{i}}{(x-a)^2}$$

Total Electric Fields:

- Perpendicular Bisector:
  - The total field is:

$$\vec{E} = \vec{E}_{-} + \vec{E}_{+} = -\frac{2kqa\hat{i}}{(y^2+a^2)^{3/2}}$$

  - Considering the case where $y >> a$ (we are very far from the dipole)

$$\vec{E} = -\frac{2kqa\hat{i}}{y^3}$$

- Along the axis ("Axial Field")
  - The total field is (a bit messy looking):

$$\vec{E} = \vec{E}_{-} + \vec{E}_{+} = \frac{4kqax\hat{i}}{(x+a)^2(x-a)^2}$$

- It simplifies greatly if we go far from the dipole, $x >> a$:

$$\vec{E} = \frac{4kqa\hat{i}}{x^3}$$

A dipole is a complex beast, and the field will vary a lot depending on where you are around the dipole. It makes sense to define a new vector, one which summarizes the properties and orientation of the dipole. To do this, we define the "dipole moment":

- The **dipole moment** points from the negative to the positive charge, and has a magnitude $p = qd$, where q is the magnitude of either of the two charges and d is the separation between the two charges. Note that $d = 2a$

In the above equations, we identify the dipole moment as

$$\vec{p} = 2qa\hat{i} = qd\hat{i} = q\vec{d}$$

This will come in handy as we think about what matter will do in the presence of an external electric field (e.g. what happens to a dipole if it's immersed in another field?).

Continuous Distributions of Charge

There is a smallest unit of charge - that carried by either of the electron or the proton. All matter is made from a large number of these particles. However, it is impractical to ask anyone to sum over 10^{23} individual point-charges to obtain the total force or field outside of the distribution.

Instead, we need to use calculus to describe such objects. In calculus, we treat the object as a sum over a large number of infinitesimal pieces, and then sum the pieces. For instance, if we have a large collection of charges, each producing a small piece $d\vec{E}$ of the total electric field $\vec{E}$ at some point P, then to obtain the total electric field at point P:

$$\vec{E} = \int d\vec{E} = \int \frac{kdq}{r^2}\hat{r}$$

Each charge element's field, $d\vec{E} = (k\,dq/r^2)\hat{r}$, so the above simply performs a sum over those elements. The strategy for solving such problems is the same: we identify the field point and the source charges - except this time the source is a continuous distribution of charge. The hard part is to find a way to express the unit vector $\hat{r}$ and distances r in terms of coordinates over which we can then integrate.

Bulk Properties

Large bodies are built from a large number of point charges. If the charge distribution is UNIFORM in the object, then we know that the amount of charge per unit of space (either line, area, or volume) is constant.

- A one-dimensional body - a line with length L - can have a LINEAR CHARGE DENSITY. This is denoted by $\lambda = \Delta q/\Delta x = Q/L$.
- A two-dimensional body - a plane with area A - can have a SURFACE CHARGE DENSITY. This is denoted by $\sigma = \Delta q/\Delta A = Q/A$
- A three-dimensional body - a volume, V - has a VOLUME CHARGE DENSITY. This is denoted by $\rho = \Delta q/\Delta V = Q/V$