

# General Physics - E&M (PHY 1308) Lecture

## Notes

### Lecture007: Potential and the Storage of Energy

SteveSekula, 8 February 2011 (created 7 February 2011)

no tags

### Goals of this Lecture

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- Discuss conductivity and lightning protection
- Learn to setup and tackle problems involving electric potential differences
- Discuss the storage of energy in an electric potential difference

### Conductors and Lightning

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- Show movie of lightning generator at Boston Museum of Science
- Discuss lightning and what it is.
- Discuss what happens when charge is deposited on a neutral conductor
- Discuss lightning protection

### The Boston Science Museum Van de Graaff Generator

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We've watched video of the person in the cage being subjected to huge bolts of electricity from the large Van de Graaff generator at the Boston Museum of Science. Let's use this as a problem we can setup and work.

The Van de Graaff consists of a conductive ball at the top which is slowly

imparted a large electric charge. Electrons are being stripped off the sphere by a belt, so the sphere acquires a positive charge. Being a conductor, all the charge migrates to the outer surface of the sphere, building up on the surface.

The **VdG** at the museum has a radius of 2.30m and develops a charge of  $Q = 640\mu\text{C}$ . Considering this device to be a single, isolated sphere of charge, let's:

- (a) find the potential at its surface,
- (b) find the work needed to bring an electron from infinity to the surface of the sphere
- (c) find the potential difference between the sphere and a point that is  $2R$  from its surface (basically, the location of the cage protecting the person).

(a) For the case of a sphere with electric charge on its surface,  $Q$ , the electric field is the same as that of a point charge carrying  $Q$  located at the center of the sphere:

$$\vec{E}_{\text{outside}} = \frac{kQ}{r^2} \hat{r}$$

Now we can evaluate the potential at the surface of the **VdG** generator sphere. But remember, **it is meaningless to speak only of the potential as an absolute thing; it's a relative thing, and we have to define the "zero" of potential in order to compute the potential difference.**

For a function that falls off AT LEAST AS FAST AS  $1/r^2$ , we see that there is one place where the field strength goes to zero: at infinity. We can therefore choose to define *zero potential for a point charge, or our sphere, as existing at  $r = \infty$* . It may seem strange to do this - that the work done to move a charge from infinity to the sphere can be finite - but keep in mind that since the field falls off as  $1/r^2$  the work required to move a charge further from the sphere becomes less and less, so we have a finite result in the end.

The electric potential difference is given by:

$$\Delta V_{AB} = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{r}$$

Let's consider the path a positive charge would take under the influence of our positively charged sphere. Our path stretches from  $r_A$  to  $r_B$ . Let us choose a simple path, since the path itself does not matter. Let us choose a straight line pointing from  $r_A$  to  $r_B$  along a line that lies along a radius of the sphere. If we define the direction  $\hat{r}$  as the direction pointing out from the center of the sphere along such a radius, then we can write  $d\vec{r} = dr \hat{r}$ , since pieces of our path will have length  $dr$  and point in the  $\hat{r}$  direction.

What about the electric field? The electric field outside of a charged spherical shell is:

$$\vec{E} = k \frac{Q}{r^2} \hat{r}$$

Thus:

$$\Delta V_{AB} = - \int_{r_A}^{r_B} \left( k \frac{Q}{r^2} \hat{r} \right) \cdot (dr(\hat{r})) = -kQ \int_{r_A}^{r_B} \frac{1}{r^2} (\hat{r} \cdot \hat{r}) dr = -kQ \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

We can now perform the integration:

$$\Delta V_{AB} = -kQ \left( -\frac{1}{r} \Big|_{r_A}^{r_B} \right) = kQ \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$

Now we can set  $r_A = R$  and  $r_B = \infty$ :

$$\Delta V_{\infty, R} = -kQ \frac{1}{R}$$

Let's see if this makes sense. Let's pretend we have a positive charge located at  $R$ , and that the charge on the sphere is positive. Thus  $Q > 0$  and  $q > 0$ . The field will cause the positive charge to move from the surface of the sphere to infinity, pushed away by the repulsive nature of the like charges. Thus we expect that  $W_{AB} > 0$  because the field will move the positive charge from a region of high-potential to a region of low potential energy. Is that true?

Let's calculate the work:

$$W_{AB} = -\Delta U_{AB} = -q\Delta V_{AB} = -q\left(-\frac{kQ}{R}\right) = \frac{kQq}{R}$$

Indeed, the work is positive! This is a good sanity check.

Now, what about the work to take a positive charge from infinity to  $R$ ? For that, we now have  $r_A = \infty$  and  $r_B = R$ . We find in this case that:

$$\Delta V_{AB} = \Delta V_{\infty,R} = \frac{kQ}{R}$$

$$W_{AB} = W_{\infty,R} = -q\Delta V_{\infty,R} = -\frac{kQq}{R}$$

In this case, the work done by the field is negative, implying that work must be done AGAINST the field to move the positive charge from infinity to the surface of the sphere. Does that make sense? The sphere is repelling the charge, so you must work against the field to bring the positive charge closer.

Plugging in our numbers to compute the potential difference for the Boston Science Museum Van de Graaf generator is:

$$\Delta V_{\infty,T} = 2.5 \times 10^6 \text{V}$$

What about for a negative charge ( $-q$ ), coming from infinity to  $R$ ? In that case:

$$\Delta V_{AB} = \Delta V_{\infty,R} = \frac{kQ}{R}$$

so nothing changes there. But, when we compute the work:

$$W_{AB} = W_{\infty,R} = -(-q)\Delta V_{\infty,R} = \frac{kQq}{R}$$

we see that it's positive, which makes sense - the field is attracting the negative charge closer to the sphere. That means that the negative charge is going from a location of high potential energy to one of low potential energy, and thus the field should be doing positive work on the charge. Indeed, that is the case here.

(b) how much work does the field need to do to move a negative electric charge from infinity to the surface of the sphere?

We've already computed the work:

$$W_{AB} = W_{\infty,R} = -(-q)\Delta V_{\infty,R} = \frac{kQq}{R}$$

We just need to plug in numbers:

$$W_{AB} = 4.0 \times 10^{-13} \text{ J}$$

(c) What is the electric potential difference between R and 2R?

$$\Delta V_{AB} = -kQ \left( -\frac{1}{r} \right) \Big|_R^{2R} = kQ \left( \frac{1}{2R} - \frac{1}{R} \right) \approx -1.3 \text{ MV}$$

This problem illustrates a few things:

- Be careful with your signs. Get your definitions right, make sure you carry through the signs of your electric charges, and you'll be steered on the right path by the math
- Having been exposed to charged spheres and other "simple" geometries, you have a powerful toolkit to begin to understand the world around you.

## Electrostatic energy

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The electric field can not only do work, but store energy. The storage of energy in electric fields, and its release, is the basis of chemical energy. The metabolizing of food and burning of fuel are basically the act of rearranging configurations of electric charge so as to release some of the energy stored in the original arrangement. The example we'll start with today seems simple, but it's deceptively powerful because it contains the essential elements of any system that stores energy in the electric field.

This exercise will not only introduce the idea of energy stored in an electric field, but will help to review some of the basic concepts we started learning a few weeks ago.

Imagine that I place a point charge,  $q_1 > 0$ , in a region of empty space free from any electric fields or other forces. How much work does it take to place  $q_1$  at a point in such a region free of electric fields and other forces?

- ANSWER: none. Absent any source of external force, we can place charge  $q_1$  at a point at no cost in energy. Remember that  $W_{AB} = \int \vec{F} \cdot d\vec{r}$  - if  $\vec{F} = 0$ , then no work is needed.

Now, let's imagine that I want to bring a second positive charge,  $q_2$ , from very far away up to a distance  $|r| = a$  from  $q_1$ . I want to figure out the work required to do that. Let's attack this from the perspective of electric potential difference:

$$\Delta V_{AB} = V_B - V_A$$

If we are going to bring another charge  $q_2$  from infinity to a distance  $a$  from  $q_1$ , then:

$$\Delta V_{r,\infty} = -kq_1/r$$

So the work required to bring that charge to a point  $a$  from  $q_1$  is:

$$W_{AB} \equiv W_2 = -q_2 V(a) = kq_1 q_2 / a$$

Now we bring in a third positive charge,  $q_3$ , also from very far away (infinity). We have to do work to get it there, against the repulsion of the other two charges. If we also place the third charge a distance  $a$  from each of the other two, forming an equilateral triangle, then we need to do work:

$$W_3 = kq_1 q_3 / a + kq_2 q_3 / a$$

So the total work required to form this configuration of charges is:

$$W = W_1 + W_2 + W_3 = 0 + kq_1 q_2 / a + kq_1 q_3 / a + kq_2 q_3 / a$$

Because electric forces are conservative forces, the work done to make the configuration is equal to the energy stored in the electric field. It takes energy to hold the charges together, to keep them from flying apart; that means energy is stored in the configuration.

Releasing any of the charges converts stored energy in the field into kinetic energy in one or more of the electric charges. Energy stored in an electric field can be released in other forms of energy, such as kinetic energy.

It doesn't matter in what order we assemble this simple example; the energy stored is the same. If one of them had been negative, it would *take work to separate that charge from the other two due to its attraction*. Electrostatic energy can be positive or negative, depending on whether or not it took work to assemble the charges in the first place.

This example is a simple metaphor for a molecule, such as a water molecule. In fact, in water the electrostatic energy is negative and it takes an injection of work (costs energy) to separate the hydrogen from the oxygen. For water, electrostatic energy is NEGATIVE. Equivalently, that is the energy released when water forms from individual atoms.

## Capacitors

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So we can store energy in an electric field. A device that does this is called a *capacitor*. Specifically,

- A **capacitor** is a pair of electrical conductors that carry equal but opposite charges

The two conductors are thus attracted to one another, and it takes work to keep them apart. The easiest to analyze capacitor is the **parallel-plate capacitor**, although capacitors come in many configurations.

Understanding the parallel-plate capacitor will give us insight into electrostatic energy and the electric field.

### Parallel plate capacitor

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A device with two parallel thin sheets of conductor. Charge is removed from one plate and added to the other (e.g. a battery can do this). Thus we have equal but opposite charges on the two plates, and close to the center of the plates we can understand the electric field by modeling the system with two infinite thin sheets of opposite charge. The field lines are perpendicular to the sheets and go from positive to negative.

Closer to the ends, the field becomes nonuniform. But we can neglect this and still get tremendously far in understanding these devices.

The electric field at the surface of the conductor is given by  $|E| = \sigma/\epsilon_0$ . If we've spread out charge uniformly over the two plates, then for either plate  $\sigma = Q/A$ , where  $A$  is the area of the plate. Thus the uniform field between the two is:

$$E = Q/\epsilon_0 A$$

Remember, these are conductors and the opposite charges accumulate on the faces closest between the two plates.

The potential difference between the two plates is

$$V = Ed = Qd/\epsilon_0 A$$



since the field is uniform.