# General Physics - E&M (PHY 1308) Lecture Notes

### Lecture 009: Using Capacitors

*SteveSekula*, 15 February 2011 (created 14 February 2011)

tags:
journal

- Discuss the energy stored in a capacitor
- Discuss how to use capacitors in practical applications
- Compare electrostatic equilibrium and non-equilibrium situations

## Energy Storage in Capacitors

How much energy can a capactor hold, given its capacitance and the electric potential difference to which it's subjected (e.g. by a battery)?

Imagine moving a small piece of charge from one plate to the other when there is a potential difference $V$ between the plates. Let's write that small piece of charge $dQ$. The amount of work required to do that is

$$dW = VdQ$$

since potential difference is work per unit charge and the work here is the work done against the field.

By moving charge $dQ$ from one plate to the other, I increase the electric field between the plates and thus increase the potential difference between the plates. From the definitions of capacitance (which doesn't change unless the geometry changes), that change in potential difference is then:

$$dQ = CdV$$

So the work involved in moving the charge is then:

$$dW = CVdV$$

Now, let's figure out what the TOTAL work involved in moving all of that charge from one plate to the other will be. We find that by adding up al the $dW$:

$$W = \int dW = \int_0^V CV \, dV = \frac{1}{2}CV^2$$

If this is the work required to move all that charge, then it's also equal to the energy stored in the electric field. Thus the electrostatic energy stored in a capacitor is:

$$U = \frac{1}{2}CV^2$$

Note: when we speak of a capacitor being "charged" or "charged up", we mean the charge on either of the plates. The net charge in the system remains zero.

## Using capacitors

Capacitors can either have nothing (vacuum) between the plates, or there can be something (a non-conductive material) between the plates. If there is vacuum (nothing), then the capacitance is what we wrote above:

$$C = \frac{\epsilon_0 A}{d}$$

If instead we stick material between the plates called a **dielectric**, we can increase the capacitance of the device. Dielectric materials are electrically neutral, but made from very many tiny dipoles. When subjected to an electric field, the dipoles align with the field and weaken it overall. When the strength of $E$ decreases for the same $Q$ as before, then $V$ decreases too. This makes capacitance, $C = Q/V$, go up.

The amount that the capacitance increases over the case without a dielectric is called $\kappa$, the **dielectric constant**. When a capacitor contains a dielectric, the capacitance is given by:

$$C = \kappa \frac{\epsilon_0 A}{d}$$

Most materials have a dielectric constant between 2-10. Air is about $\kappa = 1.0006$, while glass is $\kappa = 5.6$ and water is $\kappa = 80$.

## Connecting capacitors

Often, as in an electronic circuit, you will want to have more than one capacitor. How do you handle those situations where multiple/many capacitors are present?

### Capacitors in Parallel

The easiest case to think about and work out is when two capacitors are *in parallel* - that is, when both capacitors are arranged in such a way that they experience the same potential difference, $V$. Draw such a circuit on the board. The capacitance of the two capacitors is $C_1$ and $C_2$.

Conducting wires connect the two sides of the capacitors to "ground" (zero voltage) and $V$ (positive voltage). Let's think about the potential on just one side of the system. Since the wires of the circuit and the plates of the capacitors are all conductors, there are NO ELECTRIC FIELDS until we get to the gap between the plates. So a charge moving from the 0V side toward the plates of the capacitor feels no change in potential:

$$\Delta V_{AB} = \int \vec{E} \cdot d\vec{r} = 0$$

That is, until it reaches the gap between the plates, where the IS a uniform electric field. Here, we know that $\Delta V_{AB} = Ed$, reaching the final potential of $V$ on the other side. Then the charge would enter more conductor, where there are no electric fields. Thus the is no more change in potential after

crossing the gap.

This tells us that both capacitors feel the same potential difference when they are side-by-side (in parallel). It's like they are one big capacitor. Can we find a way to write the effective capacitance of the two capacitors, treating them as if they are one capacitor? If we could do that, then we'd just have one number to carry around with us - one capacitance.

Since V is the same for both, we have two equations relating charge and capacitance:

$$Q_1 = C_1 V$$

$$Q_2 = C_2 V$$

What if we wanted to find the total capacitance of this circuit, $C_{total}$? Well, the total charge on the system is just the sum of the charges (charges always add!):

$$Q_{total} = Q_1 + Q_2 = C_1 V + C_2 V = (C_1 + C_2)V$$

So:

$$C_{total} = C_1 + C_2$$

In fact, if have have a whole bunch of capacitors all in parallel with one another, then the total capacitance is just the sum of the individual capacitances. That's it!

$$C_{total} = \sum_{i=1}^{N} C_i \text{(capacitors in parallel)}$$

You can understand this physically by thinking about parallel plates with equal spacing placed together in parallel. Essentially, you are taking individual capacitors and adding the areas of their plates together. That's what connecting them with conductors in a parallel does. That's equivalent to just adding their capacitances.

## Capacitors in Series

What if, instead of putting capacitors side-by-side and connecting their different sides to different potentials, I places them in sequence. This is called "series" - the backend of one capacitor is connected to the frontend of the other, and so on. Draw this on the board.

The voltages are now no longer the same across the difference capacitors. What is the same? Draw the charges on the plates of one, and discuss how the charges on the plates of the other must be equal but opposite. In other words, $Q_1 = Q_2 = Q$ for two capacitors in series. We then have two equations:

$$Q = C_1 V_1$$

$$Q = C_2 V_2$$

We want to find the total capacitance. Since we have the same charge on both, we just have to sum the voltages this time. Voltages, like charges, add:

$$V_{total} = V_1 + V_2 = Q/C_1 + Q/C_2 = Q(\frac{1}{C_1} + \frac{1}{C_2})$$

$$Q = \frac{V_{total}}{(\frac{1}{C_1} + \frac{1}{C_2})}$$

From this, we realize that for capacitors in SERIES:

$$1/C_{total} = 1/C_1 + 1/C_2$$

So for capacitors in series, you have:

$$1/C_{total} = \sum_{i=1}^{N} \frac{1}{C_i}$$

The *combined capacitance is less than the individual capacitances*. Again, to think about the physics of this, take two parallel plate capacitors of equal area and plate separation. Put them in series. Now shrink the distance between them. We see that this is equivalent to a single capacitor with TWICE the spacing of the plates. Thus the total capacitance is less than the individual ones.