General Physics - E&M (PHY 1308) Lecture Notes

Lecture 011: Resistance to Electric Current

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no tags

Current Density

What if electric current is occurring in a more complex medium (e.g. NOT in a wire)? What about through a cell membrane, which isn't simply describable as a single wire carrying ions into and out of the cell. In that case, we need a more general quantity: current density, the amount of current flowing per unit area of the path.

Current density is by definition a vector, whose direction is given by the direction of current flow at a given point and whose magnitude is the current per unit area. Dividing out microscopic current equation by the area of the conductor:

$$\vec{J} = nq\vec{v}_d.$$

We see the product of drift velocity direction and charge defines the overall direction (n is always positive). So electron current densities point opposite the velocity of electrons, as per our convention that current is in the direction of positive charge motion.

As an exercise, consider an ion pump in a cell membrane. What is an ion pump? It's a protein that is capable of allowing positive ions into or out of the cell itself. That protein is embedded in the cell membrane. We can analyze the current and current density when a channel opens for 1ms and allows 1.1×10^4 singly-ionized potassium ions through ($q = +e$). The radius of the channel, treating it like a little cylinder, is $r = 0.15\text{mm}$.

- What is the current? ANSWER: $I = \Delta Q/\Delta t = 1.8\text{pA}$
- What is the current density? ANSWER: $J = I/A = 2.5 \times 10^7 \text{A/m}^2$

Conduction Mechanisms

The mechanism by which conduction really occurs is beyond the scope of this class, but we can go a long way by using the following model (which is the original model of conduction developed in the 1800s):

- charges are free to move in a conductor and respond to an electric field by moving through the conductor
- conductors are not perfect, however, and the charges suffer collisions with the atoms in the conductor

First, electric fields in conductors? Didn't we use the fact that there are no conductors in electric fields to understand the capacitor problems? Remember, we're not in electrostatic equilibrium here. With the capacitor, we had a specific situation in which charge builds up across an uncrossable gap, and eventually current stops. Here, were are considering a more general situation - when there is current, or when it doesn't stop. In that situation, we are NOT in electrostatic equilibrium and we CAN have electric fields in conductors.

Collisions cause the charges to lose energy they had gained from the electric field.These collisions provide an effective force that works against the electric field force:

$$\vec{F}_{total} = \vec{F}_{electric} + \vec{F}_{collisions}$$

That means that there is a relationship between current density and electric field that is not perfect, but somewhat diminished by the collisions. We can write this as:

$$\vec{J} = \sigma \vec{E}.$$

σ is a property of the material, and is called the conductivity of the material, and ranges between 0 and ∞ in magnitude.

This is a microscopic version of something you might have heard of before:

"Ohm's Law." Ohm's law relates the macroscopic current to the voltage. We can relate the microscopic and macroscopic laws.

First of all, for most common conductors σ is independent of electric field (a constant). Such a materials is called "ohmic". Non-ohmic materials have a conductivity that DEPENDS on electric field. The microscopic version of Ohm's law is useful in biophysics, geophysics, astrophysics, and electrical engineering, studies that deal routinely with position-dependent electric fields.

- σ = constant: Ohmic
- $\sigma(\vec{E})$: non-Ohmic

Conductivity is a measure of how well charges respond to the electric field. A perfect conductor has $\sigma = \infty$, while a perfect insulator has $\sigma = 0$. There is a related quantity, called **resistivity**:

$$\rho = 1/\sigma$$

In terms of resistivity,

$$\vec{J} = \frac{\vec{E}}{\rho}$$

Resistivity tells us how difficult it is for charge to move in a material. The higher the resistivity, the larger the electric field needed to provide the same current density. The macroscopic property of *electrical resistance* is related to resistivity.

The unit of resistivity is the $V \cdot m/A$. One V/A is called an Ohm (Ω), in honor of German Physicists Geog Ohm who explored the relationship between current and voltage. This the unit of resistivity is the $\Omega \cdot \mathrm{m}$, while units of conductivity are $(\Omega \cdot m)^{-1}$.

Resistivity and conductivity are some of the most variable properties we know about.

Material	Resistivity $\Omega \cdot \mathrm{m}$
Copper	1.68×10^{-8}
Gold	2.24×10^{-8}
Water	2.6×10^{5}
Human blood	0.70
Sea water	0.22
Silicon	23.0
Glass	$10^{10} - 10^{14}$

You can use the current density relationship to electric field to determine things like the electric field needed to drive current in your home or apartment. For instance, consider 15A of current moving through a $1.8\mathrm{mm}$ copper wire. The electric field needed to drive this current is:

$$|\vec{J}| = |\vec{E}|/\rho \longrightarrow |E| = \rho|J| = \rho\frac{I}{A} = 99\mathrm{mV/m}$$

Copper is really an excellent conductor, so it takes small electric fields to drive that current through your wiring. The value of copper in the current global market is so great that during the peak of the recent housing foreclosure crisis, foreclosed homes were broken into and stripped of copper wiring, pipes, etc. Physics defines the technological value of the material, value defines the economic incentive, and that creates markets and black markets. See? Physics *is* relevant. :-)

The macroscopic version of Ohm's Law

The original version of Ohm's law, articulated by Ohm himself, is from the modern perspective the "macroscopic" version of this law. It depends not on the microscopic properties of the conductor and charge carriers, but rather on macroscopic properties such as the electric potential difference and the overall flow of current. Ohm discovered that current the voltage could be related linearly to one another in many materials, and this is expressed as "Ohm's Law":

$$I = \frac{V}{R}$$

A given voltage can push MORE current through a material with lower resistance, R, where "resistance" is the macroscopic tendency of a material to resist the flow of current (it's related to the microscopic "resistivity," as we'll see).

Let's consider extreme cases:

- **open circuit**: this is one in which $R = \infty$. Consider a circuit with an open switch. The switch being open prevents any flow of charge - perfect resistance.
- **short circuit**: in this case, $R = 0$ and a voltage can push an arbitrary amount of current through a circuit. This can occur when, for instance, the switch in an appliance fails and all resistive load in the appliance is bypassed, causing massive amounts of current to flow through your house wiring.

DISCUSSION:

- Since all materials have some resistance, and resistance is related to collisions between atoms and charges, what might happen to house wiring under the conditions of a short circuit?

The Lessons of Ohm's Law

Ohm's law teaches us the following:

- For a fixed resistance, more voltage will drive more current
- For a low/zero resistance, a voltage can drive a huge/infinite amount of current
- For a high/infinite resistance, a voltage, no matter how large, will drive a lot less/no current

Connecting micro and macro

How do we connect our microscopic picture of conduction to the macroscopic observation that is Ohm's Law?

Imagine a conductor of area A and length L carrying an electric field $\vec{E}$ that is uniform (draw this). The electric field is related to the voltage as we have already learned:

$$V = EL$$

We know how current density relates to electric field, too:

$$J = E/\rho = I/A$$

Thus:

$$V = (\rho J)(L/q) = (\rho I/A)L$$

We can then rearrange this as follows:

$$V = \frac{\rho L}{A} I$$

We can now relate microscopic properties to macroscopic effects, and thus:

$$V = I\frac{\rho L}{A} = IR$$

So

$$R = \frac{\rho L}{A}$$

We thus learn that:

- As the area of the conductor increases for a fixed L and resistivity, the resistance drops
- As the length of the conductor increases for fixed A and resistivity, the resistance increases
- If the resistivity goes up for fixes A and L, the resistance increases
- Etc.

Resistance is connected to microscopic and geometric properties of the material.

Resistors

A **resistor** is a piece of conductor whose properties are tuned to obtain a specific resistance to the flow of current. Examples:

- heating elements in an electric stove or a hair dryer
- filament in a light bulb

Resistors are rated not only by their resistance, but also by the amount of **power that they can dissipate**.

Power

When electric current flows through a resistive material, each electron loses energy gained in the electric field through collisions with atoms in the material. This results in motions of the atoms, and this can lead to heat, light, and other forms of energy that result from the conversion of electric energy in the field (voltage) into kinetic energy of the charges, atoms, etc (vibration, etc.).

Voltage is the energy per unit charge gained by charges that "fall" through the electric potential. Current is the amount of charge per unit time that passes through a point in the material. Power is the energy per unit time dissipated by collisions in the resistor, which is just then equal to:

$$P = \frac{\Delta U}{\Delta t} = \frac{\Delta U}{\Delta Q}\frac{\Delta Q}{\Delta t} = VI$$

If we then insert Ohm's law, which relates the potential applied across the resistor to the current that can flow through it:

$$P = (IR)I = I^2 R$$

or, instead, if we are given the voltage and not the current:

$$P = V(V/R) = V^2/R$$

From either of these, we have a powerful tool that will help us understand the energy resulting from current flowing through a material in response to an electric potential (electric field).