General Physics - E&M (PHY 1308) Lecture Notes

Lecture 013: Batteries and Circuit Analysis
SteveSekula, 7 March 2011 (created 2 March 2011)

no tags

Ideal and Real Batteries

Let's take a moment to talk about batteries. We've been using an unnamed "source of electric potential difference, V" to attack these problems. An *ideal battery* delivers a steady voltage across its terminals, regardless of the current through the circuit. We denote the ability of a battery to create an electric field and do work on charge as its *electromotive force*, or EMF. We write the work per unit charge that the battery can do as $\mathcal{E}$.

Let's revisit series resistor results in lieu of this change in notation, now that we're introducing real batteries:

$$V_1 = \frac{R_1}{R_1 + R_2}\mathcal{E}$$

$$V_2 = \frac{R_2}{R_1 + R_2}\mathcal{E}$$

The battery voltage divides between the two resistors, based on their resistance. That is also why a combination of series resistors is called a **voltage divider**

- If you need to take a large source of EMF and subdivide its voltage into smaller pieces to work on more delicate equipment, you need to use series resistors to create a voltage divider.
  - Consider the kindle. In your homework, you determined that the voltage required to move *E-Ink* particles is really tiny - about 0.2mV. Why can I plug my kindle into the wall and not be afraid of the high voltage - 110V - frying my electronics? I'm protected by at

least one voltage divider built into the circuitry, tamping down the wall voltage to something more reasonable for this device.

Let's think about *real batteries* for a second.

What differentiates AAA, AA, and D-type 1.5V batteries? They all deliver 1.5V of potential difference, so why the differentiation? (Is this just a conspiracy to force me to buy different batteries for all my toys?!)

Use this discussion to motivate the discovery that different battery types can deliver different currents in the same period. D-type batteries can deliver about 3A for an hour, *AA-type* batteries 2.5A for an hour, and *AAA-type* batteries about 1A for an hour. That must mean that their internal RESISTANCES are different. According to the Energizer company's own data, *AA-type* batteries have an internal resistance of about 0.2Ω.

http://data.energizer.com/PDFs/BatteryIR.pdf

So batteries are sources of electric potential difference AND resistance. The chemical reactions that allow them to create an EMF are not steady over time (batteries wear down, and batteries can't behave perfectly for all currents). In effect, a battery is a source of EMF in series with a resistor, creating an internal voltage divider. The terminal voltage is always a bit less than the rated voltage for the battery. If the resistive load in the external circuit, R_L, is big compared to the internal battery resistance, R_{int}, then the voltage delivered by the battery is nearly $\mathcal{E}$:

$$V_{battery} = \frac{R_L}{R_{int} + R_L} \mathcal{E} \to R_{int} << R_L \to \mathcal{E}$$

Even if we short-circuit the battery (connect its terminals - ALWAYS A BAD IDEA!), we don't get infinite current because of that internal resistance. In fact, the most current we ever get is

$$I_{short} = \mathcal{E}/R_{int}$$

For a *AA-type* battery, that gives us:

$$I_{short} = (1.5\text{V})/(0.2\Omega) = 7.5A$$

How much power is that? Will that be bad?

$$P_{short} = (1.5V)(6A) = 11.3\text{W}$$

That's enough to raise the temperature of your hand, in contact with the battery, by 1-2 degrees celcius each few seconds or so. That would be noticeably hot in a very short period of time, and it's not at all good for the battery.

Kirchoff's Laws

Some circuits cannot be simplified (e.g. by combining parallel and series resistors into a single resistor). Consider the diagram in Fig. 25.13 in Wolfson. That circuit cannot be analyzed into simple series and parallel combinations. How do we tackle it?

Fundamentally, we need to apply conservation laws. The number of charges going into a part of the circuit must come out the other end, otherwise charge will build up in between and we cannot be in a steady-state situation. Kirchoff's Laws are just the realization of the application of conservation laws to the system.

Kirchoff's Laws analyze circuits in terms of *loops* - closed paths in the circuit. Why? Because if you traverse a closed path and measure things, like current, as you go, you have to return to where you started. In other words, the sum of all the changes in the energy per unit charge have to sum to zero. That is, increases and decreases in voltage must sum to zero, otherwise you're not returning to where you started.

Kirchoff's Loop Law: the sum of voltages around a closed loop is zero. This is essentially a restatement of the conservation of energy.

- The loop law holds for any closed loop in a circuit, regardless of the

complexity of the circuit. That's because fundamentally, nature always conserves energy.

In analyzing resistors in series, we observed that in steady-state the charge per unit time entering a series of resistors must be equal to the charge per unit time exiting the system. This is essentially the statement that the sum of the charges in a closed system is constant - *conservation of charge*. That means that if we consider a *node* in a circuit - a place where several conduction paths meet - the sum of the currents into the out of the node must sum to zero.

Kirchoff's Node Law: the sum of the currents at any node in a circuit is zero.

Strategies for applying Kirchoff's Laws

Let's apply these laws - essentially, conservation of energy and conservation of charge - to a multiloop circuit like the one in Example 25.4 in Wolfson. Our strategy will always be the same:

- Identify LOOPS and NODES. Label them. Do whatever you need to do to notate their existence.
- Label the currents at each node, assigning a direction to each. This is the trickiest part of the strategy, because often it's not clear where current is actually flowing and you have to think hard and carefully about your choices. Here are some rules you can follow to help you assign directions and signs:
  - For all but one node, write equations expressing Kirchoff's Node Law: the sum of the currents at any node is zero. Take currents flowing INTO the node as positive and OUT of the node as negative.
  - Write equations representing Kirchoff's Loop Law for as many independent loops as necessary. Make a decision as to whether you will go *clockwise* or *counterclockwise* around your loops, and BE CONSISTENT.
  - The voltage change going through a battery from the negative to the positive terminal is $+\mathcal{E}$; The voltage change from + to - is $-\mathcal{E}$.
  - For resistors traversed in your direction assigned to the current, the voltage change is $-IR$; for the opposite direction, it's $+IR$
  - For other circuit elements (e.g. capacitors), use the characteristics of the element to determine the voltage change. We'll come back to

that one in a bit.
- ○ You don't need equations for all loops and nodes; some will be redundant. Our example will illustrate this.
- Solve the equations for unknown quantities.
- Check your answer to see if it makes sense based on how you assigned directions.

Let's find the current in resistor R_3 in example 25.4. I choose to go counterclockwise in LOOP 1 and clockwise in LOOP 2 - that way, current in the center resistor is flowing in the same direction whether I look at LOOP 1 or LOOP 2. I accept that once I make this choice, I stick to it; even if current isn't REALLY flowing in that direction, if I stick to my definition I'll be OK.

Let's label the left loop at LOOP 1 and the right loop as LOOP 2, and the whole outer loop as LOOP 3. Let's label the top node A and the bottom node B.

- We only need to work the Node Law at one node. so let's choose node A:

$$0 = -I_1 + I_2 + I_3 \ \text{(node A)}$$

- We need loop equations for all but one loop, since two of the three overlap. Let's start with LOOP 1. Beginning at Node A, we go counterclockwise. We first encounter an EMF, which is $+\mathcal{E}_1$ because we proceed from the negative to the positive terminal. We then encounter resistor 1, which gives us a potential change of $-I_1 R_1$. We then encounter resistor 3, which gives us a potential change of $-I_3 R_3$. Thus:

$$0 = +\mathcal{E}_1 - I_1 R_1 - I_3 R_3 \ \text{(Loop 1)}$$

- We need another loop equation. Let's do LOOP 2, going CLOCKWISE in this loop and starting at node A. We encounter a battery first, which gives us a positive EMF $+\mathcal{E}_2$. We then encounter resistor 2, giving us a potential change of $+I_2 R_2$ (because we're going AGAINST the current).

We then encounter resistor 3, giving us a potential change of $-I_3R_3$. Thus:

$$0 = +\mathcal{E}_2 + (I_2R_2) - I_3R_3 \ \text{ (Loop 2)}$$

Let's plug in numbers for the known quantities in these equations:

$$0 = -I_1 + I_2 + I_3 \ \text{ (node A)}$$

$$0 = +6 - 2I_1 - I_3 \ \text{ (Loop 1)}$$

$$0 = +9 + 4I_2 - I_3 \ \text{ (Loop 2)}$$

We want I_3. We can eliminate I_1 by using the node A equation:

$$I_1 = I_2 + I_3 \ \text{ (node A)}$$

Substituting into the loop 1 equation yields:

$$0 = 6 - 2I_2 - 3I_3 \rightarrow I_2 = \frac{1}{2}(6 - 3I_3) \ \text{ (Loop 1)}$$

Finally, we substitute that into the Loop 2 equation and remove I_2:

$$0 = 9 + 2(6 - 3I_3) - I_3 \rightarrow I_3 = 3\text{A} \ \text{ (Loop 2)}$$

We had assigned I_3 an algebraic sign that was positive, flowing INTO node A. The result above - a positive current for I_3 - confirms that we inadvertantly chose the correct sign. This is actually clear from the picture - since both batteries have negative terminals adjacent to node A, current must be flowing into node A from the bottom and out the top into the batteries. If one of the batteries were reversed, the situation wouldn't be so clear and we have to rely on our math to tell us the answer.