General Physics - E&M (PHY 1308) Lecture Notes

Lecture 014: RC Circuits and Magnetism

SteveSekula, 21 March 2011 (created 7 March 2011)

no tags

Capacitors in Circuits

What happens if we add a capacitor in series with a resistor in a simple circuit? Now, for the time between activating the switch in the circuit and the time the capacitor is fully charged, current is NOT steady-state in the system; in fact, it's flowing but it's BUILDING UP on the capacitor. Eventually, this prohibits the further flow of charge in the system as the electric field in the capacitor is equal to but opposite the potential in the battery. At that point, the circuit is in ELECTROSTATIC EQUILIBRIUM and no current flows. If we then REMOVE the battery and connect the capactor across the resistor, current flows in the opposite direction as negative and positive charges rejoin each other. There is current flow at first, but then less and less later until all charges are pairs and current flow stops altogether again. This kind of circuit is known as an RC circuit. We will analyze it physically and mathematically, and draw conclusions about these systems.

Charging

Consider a battery-switch-resistor-capacitor circuit.

- Flip the switch and consider times very close to that event. Charge flows freely at first, negative charges pulled off one side the capacitor and placed onto the other.
- At first, the current through the resistor is $I = \mathcal{E}/R$. It decreases as the voltage across the capacitor increases (since $Q = CV$, as Q increases so does V across the capacitor)
- Eventually the voltage across the capacitor increases to match that of the battery
- The whole system reaches a state where the current is zero and the

capacitor is charged.

Charging a Capacitor: Kirchoff's Law View

We can analyze the circuit using Kirchoff's Loop Law. Let's go clockwise, and assume the current points in the same direction:

$$\mathcal{E} - IR - Q/C = 0$$

The equation has two unknowns - I and Q - but they are related by $I = dQ/dt$. This equation is what is known as a "differential equation" - to find Q as a function of time, we have to solve the equation. Let's take the derivative of this whole equation:

$$\frac{d}{dt}\left(\mathcal{E} - IR - Q/C\right) = 0$$

$$-R\frac{dI}{dt} - \frac{1}{C}\frac{dQ}{dt} = 0$$

Battery voltage is constant, and drops out under the derivative. Thus:

$$\frac{dI}{dt} = -\frac{I}{RC}$$

We see the differential equation revealed - it's now only in terms if I and time, and we need to solve for the function of I. We can rewrite this:

$$\frac{dI}{I} = -\frac{dt}{RC}$$

If we then integrate each side - the current side from initial to final current and the time side from 0 to a time, t - we can solve for the current as a function of time:

$$\int_{I_0}^{I} \frac{dI}{I} = -\frac{1}{RC}\int_0^t dt$$

$$ln\left(\frac{I}{I_0}\right) = -\frac{t}{RC}$$

Take the exponential of each side:

$$I/I_0 = e^{-t/(RC)}$$

$$I = I_0 e^{-t/(RC)}$$

We see that the current tends to zero, starting at an initial current I_0, after a very long time, $t >> 0$. The voltage across the capacitor at any given time, t, is given by $V_C = \mathcal{E} - V_R$. Since $V_R = IR$, then:

$$V_C = \mathcal{E}(1 - e^{-t/(RC)})$$

We see from this that the product of R and C has units of *time*, and in fact RC can be identified as a characteristic time of the circuit. RC is called the "time constant," and represents the amount of time required for the voltage to rise to 63.2% of the battery voltage:

$$V_C/\mathcal{E} = (1 - e^{-1}) = 0.632$$

Discharging the capacitor

Imagine we now remove the battery and connect the capacitor through just the resistor. This now forms a simple loop circuit. Let's assume that the current now flows from the positive side of the capacitor through the resistor. This is the direction that REDUCES the current through the resistor, so $I = -dQ/dt$. Writing Kirchoff's Loop Law:

$$0 = Q/C - IR$$

We can then take the first derivative of both sides with respect to time:

$$0 = \frac{1}{C}\frac{dQ}{dt} - R\frac{dI}{dt} = -\frac{1}{C}I - R\frac{dI}{dt}$$

$$-\frac{1}{RC}dt = \frac{dI}{I}$$

$$-\frac{1}{RC}\int dt = \int \frac{dI}{I}$$

$$-\frac{t}{RC} = ln(I/I_0)$$

$$I = I_0 e^{-t/RC}$$

$$I = V_0/Re^{-t/RC}$$

$$V = V_0 e^{-t/RC}$$

Analyzing the RC circuit: short and long times

Rather than solving differential equations, you can qualitatively think about the behavior of RC circuits (short and long times after the switch it thrown, etc.).

- Just after the battery is connected to the circuit, the capacitor acts like

a short in the circuit - charge is rapidly pulled from one side of the capactor, through the battery, and to the other side of the capacitor. After a little time passes, though, charge builds up on the capacitor and it begins to slow the rate of current; the capacitor offers more resistance to the flow of current.
- Long after the battery is connected, the voltage across the capacitor is opposite the voltage across the battery and current stops flowing. Now, the capacitor acts like a perfect open circuit, preventing current flow through it.

Draw an example circuit, with a capacitor and a resistor in parallel and discuss the circuit at early and late times.

Goals of this lecture

- Introduce magnetism and motivate its connection to the motion of electric charge
- Introduce the mathematical language needed to describe these phenomena

Demonstrations

- Permanent Magnets ("normal" and "strong")
- http://phet.colorado.edu/en/simulation/magnets-and-electromagnets
- Response of electric charge to magnetic field (try to setup cathode-ray tube and permanent magnet)

Qualitative discussion of magnetism

We will seemingly take a left turn today in the course and stop talking about electric fields for a short time. We will discuss magnetism. However, you cannot understand magnetism without understanding electric charge. You will quickly see this. We will briefly discuss the qualitative properties of magnetism and then develop the mathematics needed to describe it. We will see that something beautiful emerges: a deep symmetry between the laws that govern electric force and field and the laws that govern magnetic force and field.

- Demonstrate the existence of two "magnetic charges" - "north" and "south" - and how they are similar in behavior to electric charge (like poles repel, unlike pole attract)
- Demonstrate the change in magnetic force as a function of distance using permanent magnets and class participation.
- Demonstrate the response of electric charge to a magnetic field using a cathode-ray tube, if possible

The units of the magnetic field, denoted by $\vec{B}$ (with magnitude $|B|$), are the Tesla (T). $1\mathrm{T}$ is a large magnetic field. It's more typical to deal with fields in units of Gauss, where $1\mathrm{G} = 10^{-4}\mathrm{T}$. The earth's magnetic field has a strength of about $1\mathrm{G}$, while refrigerator magnets have strengths of $100\mathrm{G}$. MRI machines have large strengths, often several Tesla. There are stars in the universe which have collapsed, under their own gravitational pull, to compact, fast-spinning bodies called *magnetars*; the fields of a typical magnetar are about $10^{11}\mathrm{T}$.

The magnetic force on a charged particle

We have seen that a charged particle responds to a magnetic field, but not in the same way it responds to an electric field. Electric charges experience an electric force that is either parallel or anti-parallel to the electric field (depending on the charge of the particle). However, electric charges experience a magnetic force that is PERPENDICULAR to the direction of the magnetic field and to the motion of the charge. The magnetic force on an electric charge then has the following features:

- Its magnitude is proportional to the strength of the field (more field = more force), the speed of the particle (no speed = no force), and the charge of the particle (more charge = more force)
- Its direction is perpendicular to that of the motion and the field

To describe this force, we need the *cross product*. The *dot product* is a measure of how parallel two vectors are, but we need something that is a measure of how *perpendicular* two vectors are. The cross-product does this, AND has the benefit of returning a *vector* in the direction perpendicular to the original two vectors. The force law has been determined from experiment to be:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Let's review some properties of the cross-product:

- You can obtain the magnitude of the cross-product using

  $$|F| = |q||v||B| \sin\theta$$

  where θ is the angle between the velocity and the field.
- The direction of the cross-product is perpendicular to BOTH the velocity and the field. You can estimate the direction using the right-hand rule: point your fingers of your right hand in the direction of the velocity, curl your fingers in the direction of the field, and your thumb points in the direction a *positive* charge will be pushed. Flip your thumb over to get the direction a *negative* charge will be pushed. Use this to check your math.
- See updated cross-product video: http://www.youtube.com/watch?v=tqds10BFrQk (fixed an inherent math mistake in the original trick)

The full-on components of the cross-product are:

$$F_x = q(v_y B_z - v_z B_y)$$

$$F_y = q(v_z B_x - v_x B_z)$$

$$F_z = q(v_x B_y - v_y B_x)$$

Note that if the charge is negative and not positive, all components of the force reverse sign.

The total force on a charged particle: electric and magnetic force

Magnetic fields influence the motion of electrically charged particles. That suggests there is some deep connection between electricity and magnetism. That said, both electric fields and magnetic fields can be present in a region of space. In that case, the total *electromagnetic force* is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

You just add the vectors to get the total force vector on a charged particle. This property is useful; you can select particles of a specific velocity if you select the electric and magnetic fields so that only particles of that velocity can survive traveling through the fields.