General Physics - E&M (PHY 1308) Lecture Notes

Lecture 015: Magnetic Force

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no tags

Goals of this lecture:

- Discuss the forces involved with magnetic fields and currents
- Introduce the *Biot-Savart* Law

The action on an external magnetic field on electric charge

Force on a single charged particle

We saw from our experiment with the Crooke's Tube that:

$$\vec{F} \propto \vec{v} \times \vec{B}$$

In fact, the exact form of this relationship is:

$$\vec{F} = q\vec{v} \times \vec{B}$$

Review the cross product and the right-hand rule.

Cross-Product and the *Right-Hand* Rule

The cross product of two vectors returns a vector, as compared to the "dot product" or "scalar product" which returns a number. The feature of the cross product:

- $\vec{a} \times \vec{b} = \vec{c}$
- The resulting vector is *always perpendicular to the two vectors involved in the cross product*.
- The magnitude of the cross product is given by: $|c| = |a||b| \sin\theta$, where θ is the angle between vectors a and b.

The full cross product is messy to calculate, but here is a trick for doing it (show them the matrix trick).

The total force on a charged particle: electric and magnetic force

Magnetic fields influence the motion of electrically charged particles. That suggests there is some deep connection between electricity and magnetism. That said, both electric fields and magnetic fields can be present in a region of space. In that case, the total *electromagnetic force* is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

You just add the vectors to get the total force vector on a charged particle. This property is useful; you can select particles of a specific velocity if you select the electric and magnetic fields so that only particles of that velocity can survive traveling through the fields.

Units revisited

The units of magnetic field are T. A related unit is the Gauss, where $1G = 10^{-4}\mathrm{T}$. Let's figure out what Teslas are in terms of MKS units:

$$B = F/(qv \sin\theta)$$

So the units are:

$$T = \frac{\mathrm{N}}{\mathrm{C}(m/s)} = \frac{\mathrm{N}}{\mathrm{A} \cdot m} = \frac{\mathrm{N} \cdot \mathrm{s}}{\mathrm{C} \cdot m}$$

Properties of the magnetic force on a charged particle

The magnetic force always acts *perpendicular* to the velocity of the particle. Therefore, much like a ball swung at a constant speed around your head on a tether, the force alters the *direction* but not the *magnitude* of the velocity vector. It therefore causes a radial, but not a linear, acceleration. We can thus relate the magnetic force on a charged particle to the centripetal force. Let us define a positive force as acting OUTWARD from the center of rotation, and a negative force as acting INWARD toward the center of rotation. If we consider a magnetic field that is coming out of the board and completely perpendicular to the velocity $(\sin\theta = \sin(90^{\circ}) = 1$, then:

$$\vec{F}_{centripetal} = -m\frac{v^2}{r}\hat{r}$$

$$\vec{F}_{magnetic} = -q\vec{v} \times \vec{B} = -qvB\sin\theta\,\hat{r} = -qvB\hat{r}$$

Then if:

$$\vec{F}_{centripetal} = \vec{F}_{magnetic}$$

and

$$-m\frac{v^2}{r}\hat{r} = -qvB\hat{r}$$

Considering just the strength of the two forces, we can solve for the radius of the circular motion:

$$r = \frac{mv}{qB}.$$

The greater the momentum, $p = mv$, the larger the radius (the less effect on the direction of motion of the charged particle). The greater the field or the magnitude of the charge, the shorter the radius - the stronger the effect on the motion (tighter orbit).

Magnetic force on many charges: magnetic field and current

Let's step up our discussion of magnetic field, and consider a CURRENT of electric charge and not just a single electric charge. A current is just made from a whole bunch of charge carriers, each with charge q. If we imagine that these charge carriers are moving as a speed $\vec{v}$ through a conductor that contains n charge carriers per unit volume, then we can relate the total charge to the current, carrier density, and conductor products. Recall that:

$$Q_{total} = nqAL$$

Thus the total magnetic force for a conductor of length L immersed in a magnetic field $\vec{B}$ is then:

$$\vec{F} = Q\vec{v} \times \vec{B} = nqAL\vec{v} \times \vec{B}$$

Aha! But recall that:

$$I = nqAv$$

If we write the velocity vector above as the product of a number and a unit vector:

$$\vec{v} = v\hat{v}$$

Then

$$\vec{F} = nALqv\hat{v} \times \vec{B}$$

If we DEFINE a new vector, $\vec{L}$, whose magnitude is the length of the conductor and whose direction is that of the current flow ($\hat{v}$), then we can write:

$$\vec{F} = (nAqv)(L\hat{v}) \times \vec{B} = I\vec{L} \times \vec{B}$$

Practice the concept

Draw a flexible wire passing through a region of magnetic field pointing out of the board. The wire is deflected upward. Is the current to the left or to the right in the conductor?

- ANSWER: current is to the left

The Origin of Magnetism: Moving Electric Charge and the Magnetic Field Due to Moving Electric Charge

Discussion

Initiate a discussion in the class about what is magnetism, given that we observe that magnetic fields affect moving electric charge and that magnets exert forces on one another.