General Physics - E&M (PHY 1308) Lecture Notes

Lecture 020: Induction

YourName, 6 April 2011 (created 2 November 2010)

Goals of this Lecture

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- Introduce the concept of *magnetic induction*
- Demonstrate the effect and describe it mathematically

Demonstrations

- show what a moving bar magnet does to coil of wire hooked up to a current meter ("ammeter")
- show what a pendulum made from different materials does when the materials swing through a region of high, permanent magnetic field

Magnetic Induction

• Use the *PhET* demonstrator on electromagnets (<u>http://phet.colorado.edu</u> /<u>en/simulation/faraday</u>) to illustrate the concept of changing magnetic flux

The common feature in all of these experiments is *changing magnetic flux*. If we consider the field lines from the bar magnet, we see that as we move the magnet the number of flux lines enclosed in the wire loop changes over time due to the relative motion of the loop and the magnet.

The ability for *changing magnetic flux* to induce an *electric current* is known as *electromagnetic induction*.

• We have previously been exploring the phenomenon that moving

electric charge induces magnetic fields

• We now have a new phenomenon: changing magnetic fields induce electric currents (moving electric charge)

Two scientists - Michael Faraday from England and Joseph Henry from the United States - simultaneously discovered this phenomenon. The law that we use to describe this effect is known as *Faraday's Law of Magnetic Induction*.

Magnetic Flux

The change of a magnetic field near a conductor induces an electric current, and that means there must be an accompanying induced electromotive force (emf), which normally comes from a battery but now results from changing magnetic field.

We have previously explored Gauss's Law for Magnetic Fields, which tells us that the magnetic flux through a CLOSED surface will always be zero:

$$\Phi_B(closed) = \int_{surface} ec{B} \cdot dec{A} = 0$$

But what about open surfaces, like the plane of a loop of wire? In that case, it's possible to have non-zero magnetic flux. Recall the definition of flux from our discussion of electric field. Consider a vector \vec{A} that is *normal* (perpendicular) to an area (e.g. the area of a loop of wire), whose magnitude A is the area of the surface. The flux is then given by:

$$\Phi_B = \int ec{B} \cdot dec{A}$$

which is the general case when the magnetic field may depend on the location in the area where we are evaluating it. If you have a UNIFORM field and a FLAT surface, then this simplifies to:

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where θ is the angle between the normal to the area and the magnetic field direction.

Example: flux through a solenoid

Consider a solenoid with N windings whose radius is R. If we run current through the solenoid, the resulting magnetic field inside the solenoid is:

$$B = \mu_0 n I$$

where n = N/L, the number of windings per unit length of the solenoid. The field is parallel to the axis of the solenoid and thus perpendicular to the area of the solenoid. The flux of this magnetic field through the solenoid is then:

$$\Phi_B = ec{B} \cdot ec{A} = BA\cos(0) = (\mu_0 nI)(\pi R^2) = \mu_0 \pi nIR^2$$

In this example:

• the flux through the solenoid increases with current, radius, or the number of windings per unit length - which all makes sense, because all of these things serve to either increase the strength of the magnetic field OR the area of the solenoid

Faraday's Law

Michael Faraday and those that followed him discerned the exact relationship between the change in flux and the induced EMF in the circuit. Faraday's Law, along with Ampere's Law and Gauss's Laws for electric and magnetic fields, comprises three of the four fundamental laws of electromagnetism.

The law is a mathematical description of the outcome of many experiments.

$${\cal E}=-rac{d\Phi_B}{dt}$$

- Faraday's Law relates the INDUCED EMF to the CHANGE IN MAGNETIC FLUX
- The induced EMF tends to OPPOSE the change in flux, thus the presence of the minus sign. That is, the currents induced by the change in flux create their own magnetic fields which OPPOSE the change in external magnetic field.
- Flux can be changed in three ways:
 - Change the strength of the magnetic field
 - Change the area of the conductor
 - Change the orientation of the conductor and the magnetic field

(or combinations of all three)

Solving problems with Faraday's Law

- determine whether the problem has something to do with changing magnetic flux through a conductor (circuit, etc.)
- determine the expression for the flux
- differentiate the flux with respect to time, or otherwise determine the rate at which flux changes with time. This will give you the induced EMF.

Example: a changing magnetic field strength

Consider a loop of wire, radius r = 10.0cm and resistance $R = 2.0\Omega$, immersed in a magnetic field whose strength is changing by 0.10T/s. The loop is arranged so that $\cos \theta = 1$ between the field and the area. Find the induced EMF.

We have to find the flux, and then determine how it changes with time. To do this, we first compute the flux:

$$\Phi_B = BA = B(2\pi r^2)$$

We then differentiate it with respect to time. The radius is constant, so the only thing that might change with time is *B*:

$$rac{d\phi_B}{dt}=2\pi r^2rac{dB}{dt}$$

We've been given the rate of change of B with time, so we have everything we need to determine the induced EMF:

$${\cal E}=-(3.14{
m mV})$$

The magnitude of this EMF then can tell us the current in the loop:

$$I = \mathcal{E}/R = 1.6 \mathrm{mA}$$

The meaning of the sign: Induction and Energy

Let's close this lecture with a discussion of that minus sign in Faraday's Law. What does it mean?

Think about the conservation of energy. This requires that, essentially, you get nothing for free. If you do mechanical work on a magnet to move it toward a loop of wire, and then you stop pushing, you shouldn't see the magnet continue to accelerate toward the loop free of mechanical work being done on it anymore.

Let's think about what this bedrock principle means for induction.

• Let's imaging a bar magnet on the axis of a loop of wire. The bar magnet is originally at rest. Since the magnet and loop are not in relative motion, there is no change in flux and there are no currents in the loop.

• now consider giving the bar magnet a nudge toward the loop

- if the currents in the loop were to appear such that the magnetic field they create ATTRACTS the bar magnet, then the bar magnet would continue to accelerate without stop even after you stop pushing. This amounts to the loop setting up a field whose "south pole" is closest to the north pole of the bar magnet. When the bar magnet is to the left of the loop, the flux is INCREASING through the loop. Once the bar magnet gets through the loop the flux would be decreasing and the sign of the current would flip, flipping the direction of the magnetic field. Now the south pole of the loop is aimed at the south pole of the bar magnet, and it REPELS the bar magnet, causing it to further accelerate
- this would amount to something for nothing acceleration with no cost. That violates conservation of energy.
- Reality: the current in the loop that appears when the flux starts increases OPPOSES the magnetic field of the bar magnet. In other words, the field of the current loop has a north pole facing the bar magnet north pole, repelling it and slowing DOWN the magnet. The current in the wire causes heating to occur, given by $P = I^2 R$. The energy from your original push is dissipated by heating the loop, and energy is conserved.

This is called *Lens's Law*, and in short says:

"The electromotive force, and thus the current, establish by a change in magnetic flux through a conductor tends to itself establish a magnetic field that OPPOSES the change in flux."

This is the essence of the negative sign in Faraday's Law. The physics is the following:

• Faraday's law tells us that as we change the flux, the EMF is given by:

$$rac{d\Phi_B}{dt} = -{\cal E}$$

• Form Ohm's Law, we know that in a conductor (even a very good one): $\mathcal{E} = IR$. Thus

$$\frac{d\Phi_B}{dt} = -IR$$

- The current causes a magnetic field from the conductor that OPPOSES the change in flux.
- Energy is dissipated in the conductor, given by $P = IV = I^2 R$.

This explains the conductor pendulum in the magnetic field (show the demonstration again). The conductor swings down, enters a region of magnetic field. This causes a CHANGE in magnetic flux through the conductor. Currents are established in the conductor that OPPOSE the change in flux, drawing from the kinetic energy of the pendulum. That kinetic energy, which at first causes the change in flux, is dissipated as heat in the conductor as currents at first oppose the motion, then heat the conductor.