General Physics - E&M (PHY 1308) Lecture Notes

Lecture 021: Self-Inductance and Inductors

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no tags

Goals of this Lecture

- Understand "self-inductance"
- Define *induction* and see how it works in circuits

Inductance

"Inductance" is the tendency of a changing magnetic flux to induce an electric current in a circuit. There are two kinds of inductance:

- Mutual Inductance
- Self Inductance

Mutual Inductance

So far we've been considering currents induced in a conductor by an external magnetic field from, for instance, a bar magnet. But as illustrated in the *PhET* demonstrator, another current creating a magnetic field (e.g. a solenoid) that then changes in the presence of a second conductor can induce a current in the second conductor. For instance, you can move the first conductor toward or away from the second, or increase the current in the first so that its field strength changes through the second conductor.

This is known a *mutual inductance*. One conductor's changing magnetic field can induce a current in a second conductor, and the second conductors change in magnetic field can induce changes in the first.

You can transmit energy using mutual inductance (ala an electric toothbrush).

Self-Inductance

There is one more critical kind of inductance. It's so critical, it's part of the basis of modern electronics. It's called *self inductance*

In reality, it's quite simple - but it's harder to think about because it's a bit like shaking one's own hand. It can seem a bit odd.

Consider a solenoid carrying a steady current. We know that this device contains a magnetic field inside the coil, and last lecture we calculated the flux inside a solenoid. What if the current were suddenly stopped?

- QUESTION: what would happen to the magnetic field inside the solenoid?
  - ANSWER: it would try to decrease to zero.

- QUESTION: what does that do to the magnetic flux in the solenoid?
  - ANSWER: decreases it

- QUESTION: what does Faraday's Law tell us will happen as a result of a decrease in flux in the solenoid?
  - ANSWER: a current will flow in the solenoid which attempts to reinforce the decreasing magnetic flux.

The ability of a flux-carrying device, like a solenoid, to induce it's own electrical current in response to a change in that flux is called *self-inductance*.

Such devices make it harder to induced sudden changes in, for instance, current in a conductor. The flip side is also that the REMOVAL of current from such a device will leave energy stored in the magnetic field, which will then induce current even after the external voltage has been removed. These currents can be DEADLY, as they can lead to a huge induced EMF from the stored energy in the magnetic field.

Inductors

A device which is designed to exhibit self-inductance and thus resist changes in current (and therefore magnetic flux) in a circuit is called an *inductor*. Inductors are critical components in modern electronics:

- They are in all radio transmitters, as they help establish and maintain the operating frequencies of transmitters and receivers.
- They appear in audio equipment and help insure that high-frequency and low-frequency signals go to the appropriate speakers (tweeters and woofers) in the system. This is how you get the right sound to the right speaker to get a crisp and authentic audio experience.

A simple example of an inductor is a solenoid. It contains many turns and the magnetic field from one side of the coil can interact with conductor on the other side of the coil, providing conditions for self-inductance.

Inductance

Consider a device placed in a circuit which has a magnetic field and that magnetic field penetrates parts of the device, yielding a potential self-inductance.

As long as the current in an inductor is steady, there is no change in magnetic flux. Thus *steady state* circuits lead to situations where the inductor makes no opposition to the circuit. In the steady state, when current flow is constant, the inductor acts like any other conducting part of a circuit.

But when the current in the circuit CHANGES, the inductor will act to oppose that change. If the current is flowing down through an inductor, and suddenly begins to decrease, the self-inductance will create an EMF in the inductor that forces current in the original direction of flow, opposing the change.

We can describe this mathematically. The flux in a device that contains a magnetic field through its own area is proportional to the current in the device. More current = more flux, less current = less flux. This proportionality is JUST LIKE Ohm's Law.

Recall that Ohm's Law related the voltage in a circuit the to current through a constant of proportionality: the resistance.

$$V = RI$$

In an inductor, flux is related to current through another constant of proportionality: the *inductance*:

$$\Phi_B = LI$$

Inductance of a Solenoid

A solenoid has an interior magnetic field created by current in the wire. That magnetic field is given by:

$$B_{inside} = \mu_0 n I$$

where n is the number of turns per unit lenth. What is the flux inside the solenoid due to the self-induced magnetic field?

$$\Phi_B = BA_{total}$$

The total area here is the area of each loop (A) in the solenoid, multiplied by the number of loops $N = nL$. Thus:

$$\Phi_B = (\mu_0 n I)(nLA) = \mu_0 n^2 ILA$$

To obtain the inductance of the solenoid, we divide by the current:

$$L_{solenoid} = \frac{\Phi_B}{I} = \mu_0 n^2 LA$$

Inductance and induced voltage

From Faraday's Law, we know that:

$$\mathcal{E}_L = -\frac{d\Phi_B}{dt}$$

Substituting $\Phi_B = LI$ into this equation:

$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Inductors in circuits

Analyze a simple battery, resistor, inductor circuit. The Kirchoff's loop law equation for this circuit is:

$$\mathcal{E}_0 - IR + \mathcal{E}_L = 0$$

If we then take the time derivative of this equation to see how it changes as the current and inductor EMF change:

$$-R\frac{dI}{dt} + \frac{d\mathcal{E}_L}{dt} = 0$$

We can then substitute for $\frac{dI}{dt}$ using the inductance version of Faraday's Law:

$$-R\frac{-\mathcal{E}_L}{L} + \frac{d\mathcal{E}_L}{dt} = 0$$

or in other words:

$$\frac{d\mathcal{E}_L}{dt} = -\frac{R}{L}\mathcal{E}_L$$

This is another one of those first-order differential equations we solved when looking at capacitors in circuits. The solution is very similar:

$$\mathcal{E}_L = -\mathcal{E}_0 e^{-Rt/L}$$

Before the battery is connected, current is zero. The battery is connected and current is established in the wire. But the self-inductance of the inductor creates a "back EMF" that OPPOSES this current, and so at first no current will flow in the circuit. As the magnetic flux change decreases a short time later, more current begins to flow in the circuit. As the current approaches steady-state, the magnetic flux in the inductor changes less and less until it becomes steady, and the inductor voltage drops to zero.

Energy storage in magnetic fields

Much like capacitors were the circuit element that helped us think about where energy is stored in the capacitor (in the electric field), inductors are the circuit elements that help us understand where energy in the inductor is stored. Let's consider the power in this circuit and how it is delivered to or dissipated by the circuit.

Begin with Kirchoff's loop law equation again:

$$\mathcal{E}_0 - IR - L\frac{dI}{dt} = 0$$

We want to understand power in the circuit, so multiply the whole equation by the current:

$$I\mathcal{E}_0 - I^2R - LI\frac{dI}{dt} = 0$$

The first term is the power *delivered to the circuit by the battery*. The second term is the power *dissipated by the resistor*. The third term is the *power dissipated by the inductor*. We can write that:

$$P_L = LI\frac{dI}{dt}$$

Let's see how the energy stored in the inductor is related to the inductance:

$$P_L = \frac{dU_L}{dt} = LI\frac{dI}{dt}$$

Multiply both sides by dt and integrate both sides:

$$\int dU_L = \int LIdI$$

$$U_L = \frac{1}{2}LI^2$$

Recall that the energy stored in the ELECTRIC field of a capacitor was:

$$U_C = \frac{1}{2}CV^2$$

Very similar! Another interesting symmetry between electric and magnetic fields.

What about energy density stored in the magnetic field? Consider the solenoid, whose inductance can be easily determined:

$$L_{solenoid} = \mu_0 n^2 AL$$

Plugging this into our energy equation and dividing by the volume of the solenoid, $VOL = AL$:

$$u_B = \frac{U_L}{VOL} = \frac{1}{2}\frac{\mu_0 n^2 AL}{AL}I^2 = \frac{1}{2}\mu_0 n^2 I^2$$

We can further simplify this by recalling that $B_{solenoid} = \mu_0 n I$, so that $B^2/\mu_0 = n^2 I^2$, and thus:

$$u_B = \frac{1}{2}\frac{1}{\mu_0}B^2$$

This equation, it turns out, holds true for any magnetic field. The energy stored in a volume is simply related to the magnetic field strength (squared) and constants.