General Physics - E&M (PHY 1308) Lecture Notes

Lecture 024: Mirrors and Thin Lenses

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no tags

Goals of this lecture

- introduce the concepts of "real" and "virtual" images
- discuss the images that result from mirrors
- discuss the images that result from simple lenses

Concepts: Real and Virtual Images

As we have seen in the previous lecture, **reflection** and **refraction** alter the propagation of light following well-defined laws. Technology such as microscopes, telescopes, cameras, contact lenses, and your own eyes relies on these two effects to form **images** that provide a visual representation of reality.

We will now study a subject called "geometrical optics" - the investigation of image formation when the wavelength of the light used to form an image is MUCH SMALLER than the object being imaged. In the last formal lecture of the course, we'll discuss what happens when that is no longer the case.

There are two kinds of images:

- **real images**: these are formed when light comes directly from the image to our eyes.
- **virtual images**: these are formed when light only appears to come from the location of the image.

Demonstrate real and virtual images using a plane mirror imageand the image of a quarter from a parabolic reflector.

Images with Mirrors

Plane Mirrors

A plane mirror is a flat, highly (perfectly) reflective surface that, when light rays reach the surface, object the law of reflection we discussed last time:

$$\theta_1 = \theta_1'$$

where the subscript denotes the medium in which light is propagating before and after the reflection (e.g. air, water, etc.). Consider the image of a clock from the back of the classroom in a plane mirror.

Let's imagine that we have a vertical arrow standing in front of the mirror, and we wish to use "ray tracing" - following reflected rays of light - to locate points on the virtual image. For instance, what if we wanted to locate the top of the arrow in the virtual image? We only need *two lines to locate a point* - this is a tenet of geometry. For instance, if I want to locate a point in space I need only the intersection of two lines to do this.

We can use this geometric principle and some ray tracing to figure out where the top of the arrow virtual image will be located. By following two rays emitted from the real top of the arrow, we can point back into the mirror along the path of the reflected rays and find the intersection of the rays in the virtual image. This intersection tells us where the top of the arrow in the virtual image will be located "behind" the mirror.

Plane mirrors preserve an object's length and upright orientation, but they do so by reversing front and back (not left and right - a common misconception). Consider if you were to point straight at the mirror. Your virtual image is pointing not in the same direction, but in the opposite direction. Plane mirrors reverse coordinate axes that are perpendicular to their surface. If the positive z-axis for the real you points into the mirror's surface, then the positive z-axis of your virtual image points OUT of the surface of the mirror in exactly the opposite direction.

Curved Mirrors

In contrast to plane mirrors, **curved mirrors** form images that may be upright or inverted, virtual or real, large or small.

Parabolic mirrors make the best reflectors. Why is this? The parabola has a special property: a line parallel to the parabola's axis makes the same angle to the normal of the parabola surface as does a second line drawn to a special point known as the **focus** or **focal point** of the parabola. In fact, all reflected rays in a parabolic reflector will converge to this single point. That makes a parabola excellent for concentrating diffuse parallel rays down to an intense spot, or conversely taking a diffuse light source located at the focus and generating a parallel beam of light.

Close to the apex of the parabola, it's nearly indistinguishable from a sphere. Thus *spherical mirrors*, which are cheaper and easier to manufacture, are often substituted for parabolic mirrors. However, the approximation of a spherical to a parabolic surface is only an approximation, and one has to be careful or you risk manufacturing a mirror that doesn't focus entirely at the expected focal point.

The problem of a spherical mirror that doesn't focus at the parabolic focal point is called *spherical aberration* - when the approximation to a parabola just isn't good enough. If such distortions become bad enough, they can endanger a scientific mission (e.g. The Hubble Space Telescope).

Taking the sphere as an approximation to a parabola, we can think about what images will look like when reflecting off the surface of such a mirror:

- Consider a spherical mirror whose center is labeled C (this is the point where all the radii meet), whose focus is labeled F (where rays that enter parallel to the mirror axis will then be reflected.

    - Rays that travel parallel to the mirror axis will be reflected through the focus
    - Rays that travel through the focus will be reflected parallel to the axis
    - Rays that travel through the center will reflect back along their incident trajectory (since lines that go from the center, C, to the mirror surface travel on radii, which means $\theta_i = 0$.
    - Rays that strike the center of the mirror are reflected symmetrically about the axis

Consider the following scenarios:

- An object located past C and F, represented by a vertical arrow:
  - Consider a ray that leaves the point of the arrow and travels through F. It must reflect parallel to the axis
  - Consider a ray that leaves the point of the arrow and travels parallel to the axis. It must reflect through F.
  - Where the reflected rays cross is the location of the image. Is it real or virtual?
    - We see that rays appear to come from the image to an observer. Thus this image is REAL. The image is INVERTED, however, and located between the original and the mirror. It is also REDUCED in size compared to the original object.

- An object located between C and F, again represented by a vertical arrow
  - Do ray tracing for an incident parallel and incident focal ray. Show that the image that results is REAL, ENLARGED, INVERTED.

- An object located between F and the mirror.
  - Again, do ray tracing for an incident parallel and incident focal ray. Show that the image is VIRTUAL, since the only place the reflected rays can meet is behind the mirror. The image is also UPRIGHT and ENLARGED.

Try to demonstrate the above with a camera and a parabolic/spherical reflector.

Convex Mirrors

The above mirror types are called *CONCAVE* since the light is incident to the inside of the curvature. If we flip such a reflector around and let light strike the outside of the curvature, we have a *CONVEX* mirror.

The focus has less obvious meaning for such a mirror, but its position still controls the optics.

- Place an object to the left of a concave surface
  - Ray trace a ray that is incident parallel to the axis and incident at such an angle that it would strike the focus if it could penetrate the mirror. Show that the resulting image is always virtual, upright, and reduced in size.

The Mirror Equation

Ray-tracing is a good way to build an intuitive feel about images. For instance, making a sketch like those above will help you to attack an optics problem and see if the image is real or imaginary.

But if you have to calculate, you need to use the mirror equation.

We begin by defining the **magnification** as the ratio of the heights of the object and image. This can be written as :

$$M \equiv \frac{h'}{h}$$

where h' is the height of the image and h is the height of the object. We see that this is a sensible definition; if the image appears LARGER than the object, we have *magnified* it (made it bigger) and $|M| > 1$. If the image is SMALLER than the object, we have reduced it and $|M| < 1$.

In addition, for an object that is to the left of the center of the curvature the image is inverted, so the h' has the opposite side of h. Thus here the magnification will be negative.

Consider now two kinds of rays from the object: incident parallel, and incident on the mirror center . The latter of these will reflect symmetrically around the mirror axis. Thus the triangle created by considering the object top, mirror center, and object bottom is *similar* (in the trigonometric sense - the internal angles of the two triangles are identical, and so the ratio of their sides are also identical).

Thus:

$$M = \frac{h'}{h} = -\frac{s'}{s}$$

where s' is the length of the base of the image triangle and s is the length of the base of the object triangle. Distances along the axis inside the curvature are positive, while those on the convex side are negative.

If we consider the ray that reflects through the focus instead, we see from that triangle that:

$$-\frac{h'}{h} = \frac{(s' - f)}{f}$$

where f is the distance from the mirror center to the focal point. We want to locate the position of an image given the position of the object and the focal length of the mirror. These two equations allow us to relate s, s', and f:

$$\frac{(s' - f)}{f} = \frac{s'}{s}$$

If we do a little algebra, we will find:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

The is the Mirror Equation.

Applying the Mirror Equation

Although we derived this for a real image, it applies just as well if the image is virtual. For instance, what if $s < f$ (the object is between the focal point and the center of the mirror)? Then:

$$\frac{1}{f} - \frac{1}{s} = \frac{1}{s'}$$

Since $s < f$, $1/s > 1/f$. So the left-hand side of the above is negative, and we find the image lies on the other side of the mirror - the side where $s' < 0$. That's where virtual images occur, and this equation works. You can work the trig to see that this is true.

If you know s and s', or s' and f, you can determine the magnification AND you can figure out how high the image will be. It's a simple, but VERY POWERFUL equation, and all from considering light as a ray that can reflect according to the law of reflection.