

General Physics - E&M (PHY 1308) Lecture

Notes

Lecture 025: Lenses

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no tags

Goals of this Lecture

- Discuss what rays do when passing through a thin lens
- Discuss more complex lenses

Lenses: definitions

The following terminology applies to lenses and builds on concepts we developed when discussing mirrors:

- A **lens** is a transparent piece of material that uses refraction to form images.
 - unlike a mirror, light passes *through* a lens to form an image
 - *convex* lenses, unlike *convex* mirrors, focuses parallel rays to a focal point and is therefore a *converging lens*. *convex lenses* have positive focal lengths that lie on the side of the lens **OPPOSITE** the object.
 - *concave* lenses, in contrast to *concave mirrors* and *convex lenses*, refracts parallel rays so that they appear to *diverge* from a focal point - thus, they are diverging lenses. They have focal lengths that are negative and lie on the same side as the object.

A good demonstrator of lens principles is:

http://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Thin Lenses

We begin by exploring *thin lenses* - those whose radius of curvature is extremely large compared to the thickness of the lens. Although light will actually refract TWICE - once at the air-material interface and again at the material-air interface. However, for a thin lens the thickness is so small that it suffices to approximate the process as leading to only ONE refraction (one bending) as it crosses the center-plane of the lens.

Unlike a mirror, light can pass through the lens in either direction; it therefore has two focal lengths. For a thin lens, the focal length will be the same on either side of the lens. Thus for a thin lens, it doesn't matter whether light passes one way or the other through the lens.

Ray Tracing in Thin Lenses

As with mirrors, only two rays are needed to pinpoint the location of an image. We can consider the following two principal rays:

1. Parallel Incident Rays: these will refract through the lens and converge at the focal point of the lens.
2. Incident Apex Ray: a ray that passes through the physical center of the lens will emerge from a thin lens undeflected from its original path; thus such a ray passes straight through the lens.

Let's consider a few object locations and show them in the thin lens demonstrator:

1. An object that is located further than $2f$ (2-times the focal length)
 - a. this leads to an inverted real reduced image on the other side of the lens located between f and $2f$ from the lens.
2. An object that is located between f and $2f$
 - a. this leads to an inverted, real, enlarged image on the other side of the lens. That image is located beyond $2f$ from the lens
3. An object that is located between f and the lens (within the focal length)
 - a. this leads to an upright, virtual, enlarged image on the same side as the object

Virtual images are visible only when looking through the lens. A good example of such a case is a magnifying glass, which you place above text or an image at a height that is within the focal length of the lens; you then see

an upright, enlarged image of the phonebook behind the lens.

The Lens Equation

We can get quantitative and derive the lens equation. Consider again pairs of triangles formed by the object/image system for a convex lens for two different principal rays (parallel and apex incident rays).

We can again define the magnification of the lens as the ratio of the heights of the object and image,

$$M = \frac{h'}{h}$$

Considering incident apex rays we have two similar triangles on either side of the lens and find:

$$-h'/h = s'/s$$

Considering incident parallel rays and the inverted image with height $-h'$, we find two similar triangles and we can relate sides as follows:

$$-h'/(s' - f) = h/f \longrightarrow -h'/h = (s' - f)/f$$

We can then do some algebra and find the relationship between object distance, image distance, and focal length for a thin lens:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

This equation was derived for a real image, but it holds for virtual images where:

- $s' < 0$ (virtual image)

and for the case where the lens is concave and thus diverging:

- $f < 0$ (diverging lens)

Refraction in lenses: the details

We now move to consider *thick lenses* where the thickness of the lens is comparable to the radius of curvature of the lens. This class of lenses also includes cases where the lens curvature is different on the two sides of the lens, leading to differing focal lengths on the two sides.

This is a more general treatment of lenses.

Refraction at a curved surface

Consider two materials: an outer medium n_1 (e.g. air) and a transparent refracting medium n_2 that comprises the curved surface. Consider a number of rays incident on the lens medium at small angles to the normal to the surface. We can quickly show that all such rays are focused to a common point, independent of their incident angle from the axis of the lens.

Let's consider such a case, where a ray at a small angle α to the axis is incident on the lens at a small angle θ_1 to the normal to the surface. The distance from the origin of the ray to the lens surface along the axis is s , and the refracted ray strikes a point on the axis inside the lens a distance s' from the lens surface. The refracted ray makes an angle γ with the axis. The normal makes an angle β with the lens axis inside the material.

We can apply Snell's law at the interface:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For small angles, $\sin x \approx \tan x \approx x$.

Thus Snell's Law becomes:

$$n_1 \theta_1 = n_2 \theta_2$$

Considering triangles we find that $\theta_2 = \beta - \gamma$ and $\theta_1 = \alpha + \beta$. Then we have Snell's Law:

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

In the small angle approximation, the arc BA is so close to a straight line that we can write:

$$\alpha \approx \tan \alpha \approx BA/s$$

Similarly,

$$\beta \approx BA/R$$

and

$$\gamma \approx BA/s'$$

Thus we have Snell's Law:

$$n_1 \left(\frac{BA}{s} + \frac{BA}{R} \right) = n_2 \left(\frac{BA}{R} - \frac{BA}{s'} \right)$$

Canceling out BA on all sides:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

The incident angle α appears nowhere in this equation. Thus this equation holds for ANY small angle α . Light comes to a common focal point inside the lens.

This equation is not just useful for real images:

- It applies to virtual images, $s' < 0$
- It applies to concave surfaces if we take $R < 0$
- It even works for a FLAT surface, if we take $R = \infty$. Thus it can be applied to thick plate glass windows, for instance.

Lenses: thick and thin

We can now apply this more generic version of the lens equation to thick and thin lenses. Let's consider a thick lens, surrounded by air $n_1 = 1$, with two different curvature radii on the two sides of the lens.

We can treat thick lenses in two stages:

1. Light enters the lens from one side from the object at position O. The light refracts on the surface with curvature R_1 and forms an image I_1 . We can use the above equation to find that image location
2. Light from image I_1 then exits the other side of the lens, with curvature R_2 . It refracts at that surface and forms a second image, I_2

Treating the two halves of the problem, for the left side (image 1):

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1}$$

Image 1 becomes object 2, and $s_2 = t - s'_1$ (where t is the thickness of the lens). Applying the lens equation again:

$$\frac{n}{t - s'_1} + \frac{1}{s'_2} = \frac{1-n}{R_2}$$

The Thin Lens Approximation

If we then make this lens thin, such that $t \ll R_1$ and $t \ll R_2$, this means essentially sending the thickness of the lens to zero. We then have:

$$\frac{1}{s_1} + \frac{n}{s'_1} = \frac{n-1}{R_1} \text{ (left side)}$$

$$-\frac{n}{s'_1} + \frac{1}{s'_2} = \frac{1-n}{R_2} \text{ (right side)}$$

Adding the two equations, we obtain:

$$\frac{1}{s_1} + \frac{1}{s'_2} = \left(\frac{1}{R_1} - \frac{1}{R_2} \right) (n-1)$$

The left-hand side here is equivalent to the left-hand side, and we can drop the primes and subscripts because the equation only contain the object distance and the FINAL image distance. Thus the right-hand sides must be equivalent:

$$1/f = \left(\frac{1}{R_1} - \frac{1}{R_2} \right) (n-1)$$

This is the *lensmaker's formula* and tells you how to make a lens with a desired focal length from a material n and the appropriate grinding of the two sides of the material to make curved surfaces with radii R_1 and R_2 . The radii can be positive or negative; they are positive when a convex surface faces the incident light, and negative when a concave surface faces the incident light.