

General Physics - E&M (PHY 1308) Lecture

Notes

Clarification: electric potential around a sphere

[SteveSekula](#), 8 February 2011 (created 8 February 2011)

no tags

I wanted to provide a step-by-step write-up of the derivation of the electric potential difference between infinity and the surface of a charged sphere. By doing this, I hope I will clarify some of the questions of signs that arose during Lecture 7.

The electric potential difference of a charged sphere

- My mistake: when I setup the problem, I considered a path of a charge from infinity to the surface of the sphere. That is the path a *negative* charge would take in the field of the sphere. That means that, implicit in the derivation was that the charge starting at infinity is a NEGATIVE charge.
- Corrected derivation: It is better not to hide signs in a derivation - instead, it is better to start by considering what positive charges would do and then, later, when we need to think about a positive charge we can change $q \rightarrow -q$. The derivation below takes this strategy, and doesn't hide the signs.

First, let us remind ourselves of the question. We have a large sphere of radius $R = 2.3\text{m}$. Its surface carries a total charge $Q = 640\mu\text{C}$. We want to find the potential difference between the surface of the sphere ($r_A = R$) and a point very, very far away ($r_B = \infty$).

The electric potential difference is given by:

$$\Delta V_{AB} = - \int_{r_A}^{r_B} \vec{E} \cdot d\vec{r}$$

Our path stretches from r_A to r_B . Let us choose a simple path, since the path itself does not matter. Let us choose a straight line pointing from r_A to r_B along a line that lies along a radius of the sphere. If we define the direction \hat{r} as the direction pointing out from the center of the sphere along such a radius, then we can write $d\vec{r} = dr \hat{r}$, since pieces of our path will have length dr and point in the \hat{r} direction.

What about the electric field? The electric field outside of a charged

spherical shell is:

$$\vec{E} = k \frac{Q}{r^2} \hat{r}$$

Thus:

$$\Delta V_{AB} = - \int_{r_A}^{r_B} \left(k \frac{Q}{r^2} \hat{r} \right) \cdot (dr(\hat{r})) = -kQ \int_{r_A}^{r_B} \frac{1}{r^2} (\hat{r} \cdot \hat{r}) dr = -kQ \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

We can now perform the integration:

$$\Delta V_{AB} = -kQ \left(-\frac{1}{r} \Big|_{r_A}^{r_B} \right) = kQ \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

Now we can set $r_A = R$ and $r_B = \infty$:

$$\Delta V_{\infty, R} = -kQ \frac{1}{R}$$

Let's see if this makes sense. Let's pretend we have a positive charge located at R, and that the charge on the sphere is positive. Thus $Q > 0$ and $q > 0$. The field will cause the positive charge to move from the surface of the sphere to infinity, pushed away by the repulsive nature of the like charges. Thus we expect that $W_{AB} > 0$ because the field will move the positive charge from a region of high-potential to a region of low potential energy. Is that true?

Let's calculate the work:

$$W_{AB} = -\Delta U_{AB} = -q\Delta V_{AB} = -q \left(-\frac{kQ}{R} \right) = \frac{kQq}{R}$$

Indeed, the work is positive! This is a good sanity check, and kudos to those of you who pointed this out during class. For me, as for you, this check is crucial in making sure you've setup the problem correctly.

Now, what about the work to take a positive charge from infinity to R ? For that, we now have $r_A = \infty$ and $r_B = R$. We find in this case that:

$$\Delta V_{AB} = \Delta V_{\infty,R} = \frac{kQ}{R}$$

$$W_{AB} = W_{\infty,R} = -q\Delta V_{\infty,R} = -\frac{kQq}{R}$$

In this case, the work done by the field is negative, implying that work must be done AGAINST the field to move the positive charge from infinity to the surface of the sphere. Does that make sense? The sphere is repelling the charge, so you must work against the field to bring the positive charge closer.

What about for a negative charge ($-q$), coming from infinity to R ? In that case:

$$\Delta V_{AB} = \Delta V_{\infty,R} = \frac{kQ}{R}$$

so nothing changes there. But, when we compute the work:

$$W_{AB} = W_{\infty,R} = -(-q)\Delta V_{\infty,R} = \frac{kQq}{R}$$

we see that it's positive, which makes sense - the field is attracting the negative charge closer to the sphere. That means that the negative charge is going from a location of high potential energy to one of low potential energy, and thus the field should be doing positive work on the charge. Indeed, that is the case here.

Assembling charges

In the example where we now take three positive charges and assemble them in close proximity (thus storing energy in their electric fields by

placing them in this configuration), I needed to be more careful about my definitions of work.

- My mistake: I mistook the work we do bringing a charge in from infinity for the work done by an electric field doing the same thing. I should have been more careful. W_{AB} in the discussion of electric potential difference is the *work done by the field*. The work done AGAINST any field is always $-W_{AB}$.

- The correction: with this definition in mind, we need to compute the work WE do when bringing a positive charge q from a point r_A to a point r_B . If the work the field does to make this happen is

$$W_{AB} = -kQq \left(\frac{1}{r_B} - \frac{1}{r_A} \right), \text{ then the work WE do against the field is}$$

$$W = -W_{AB} = kQq \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$$

We bring the first charge in from very far away (infinity) and place the charge, $+q$, anywhere we like. It takes no work to do this since there are no other charges around. Thus the work to bring in this first charge, $q_1 = +q$, is:

$$W_1 = 0$$

Now, we want to bring a second charge $q_2 = q$ in from infinity to a point that is a distance $r_B = a$ away from the first charge. The work required to do this is:

$$W_2 = kQq \left(\frac{1}{r_B} - \frac{1}{r_A} \right) = kQq \frac{1}{a}$$

Finally, let's bring in a third charge $q_3 = q$ from infinity and place it a distance a from q_1 and an equal distance a from q_2 . The work required to do this is the sum of two works: the work we do against q_1 and the work we do against q_2 independently. Thus:

$$W_3 = kQq \frac{1}{a} + kQq \frac{1}{a}$$

Since I've chosen all the charges to have the same magnitude, the total work we have to do to assemble this is:

$$W_{total} = W_1 + W_2 + W_3 = 3 \frac{kQq}{a}$$