Modern Physics (PHY 3305) Lecture Notes

HomeworkSolutions001

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Point Distributions

no tags

Points were distributed as follows for each problem:

Problem	Total	Point Distribution
SS-1	10	Part 1: correct application of derivative to formula for E '(3 points)' Part 2a: Correct application of substitution for F '(2 points)'; Correct application of chain rule '(2 points)' Part 2b: correct application of integral to function or use of substitution from Part 1 '(3 points)'
SS-2	10	Part 1: drawing meets requirements in problem '(2 points)', is labeled correctly '(1 points)', and is a correct representation of the function '(2 points)' Part 2: correct application of the second derivative '(3 points)' and correct answer '(2 points)'
CH2-2	5	Recognized that although emission and reception of the light signals is simultaneous for Anna, a stationary observer sees the receipt of the signals (not their origin) as non-simultaneous '(5 points)'
CH2-3	5	Recognized that there are two speeds, but since they only differ by the reported direction in each frame, there is no need to have two distinct speeds '(5 points)'
CH2-4	5	Part a: applied either the proper time argument or the Lorentz Transformation correctly to the hang-glider '(2 points)' Part b: applied either the proper time argument or Lorentz Transformation correctly to the people on the ground '(2 points)' Part c: Commented on whether reconciliation was needed '(1 Point)'

CH2-20	10	Applied either the Lorentz Transformation or proper time argument '(7 points)', and obtained the correct answer '(3 points)'
CH2-21	10	Applied either the Lorentz Transformation or proper time argument '(7 points)', and obtained the correct answer '(3 points)'
CH2-22	15	Part a: Applied either the Lorentz Transformation or proper length/time argument '(7 points)', and obtained the correct answer '(3 points)' Part b: Applied either the Lorentz Transformation or proper time argument '(3 points)', and obtained the correct answer '(2 points)'
CH2-24	15	Part a: Applied Lorentz Transformation or Proper Length argument '(2 points)' and obtained correct answer '(2 points)' Part b: recognized for whom the measurement of the ends of the pole must be simultaneous '(3 points)' Part c: recognizes that length contraction of the barn applies for the pole vaulter '(3 points)' Part d: Applied Lorentz Transformation to the problem '(3 points)' and obtains correct answer '(2 points)'
SS-4	15	Part 1: Applied Binomial Transformation '(2 points)' and obtained the correct answer '(1 point)' Part 2: recognized the relationship between uncertainty in time and uncertainty in distance for light signals '(3 points)' and obtained correct answer '(1 point)' Part 3: applied time dilation and the binomial expansion '(2 points)' and obtained the correct answer '(1 point)' Part 4: provided a clear argument based on the answers to the previous questions '(3 points)' Part 5: provided a clear argument based on the answers to the previous questions '(2 points)'

Deductions outside of the above:

- 1 point is deducted for failing to box the numerical answer

- 1 point is deducted for incorrectly applying significant figures
- other points are deducted as outlined in the homework policy

MATH AND PHYSICS WARM-UPS

Problem *SS-1* *(10 Points)*

1. Calculate the first derivative with respect to u (that is, dE/du) of the following equation (representing the kinetic energy of a body):

$$E = \frac{1}{2}mu^2$$

SOLUTION
The first derivative of a generic function of $f(u) = u^n$, where n is an integer, is given by

$$\frac{df(u)}{du} = nu^{n-1}.$$

In this specific case, $f(u) = E = \frac{1}{2}mu^2$, so the derivative is given by

$$\frac{df(u)}{du} = \frac{dE}{du} = \frac{d}{du}\left(\frac{1}{2}mu^2\right) = \frac{1}{2}m\frac{d}{du}\left(u^2\right)$$

$$= \frac{1}{2}m(2u) = mu$$

The solution, therefore, is

$$\frac{dE}{du} = mu$$

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2. When a force is applied to an object along the x direction, it is displaced by an amount dx. The *work* (W) done on the object is given by the equation

$$W = \int F\,dx.$$

The force applied to the object can also be written as the change in momentum with respect to time, $F = dp/dt$, where momentum is defined as the product of the mass and velocity, u, (along the x direction) of the object at any given moment, $p = mu$. The application of the force therefore changes the velocity of the object, where velocity $u = dx/dt$.

   a. Show that the definition of the work done on the object can be rewritten as:

$$W = \int F\,dx = \int mu\,du.$$

[HINT: review the "chain rule" of calculus. If a variable x is a function of y, and y is a function of z, you can change variables in a derivative, such as dx/dy, from y to z by expanding the derivative to $dx/dy = (dx/dz)(dz/dy)$ Also remember that $(dx/dz)dz = dx$]

SOLUTION
Let us begin by thinking about how all of the information in this problem is related. We begin by writing down the integral that we are asked to evaluate:

$$W = \int F\,dx.$$

We know that force and momentum are related by $F = \frac{dp}{dt} = m\frac{du}{dt}$. We can substitute that into the problem to find

$$W = \int \left(m\frac{du}{dt} \right) dx.$$

Now we need to apply the chain rule, because we have an integral over one variable (x) and a derivative with respect to another (t). The chain rule, applied to this problem, lets us write

$$\frac{du}{dt} = \frac{du}{dx}\frac{dx}{dt}.$$

We can now substitute into our integral, swapping the order in which the two distinct derivatives are written:

$$W = \int m\frac{dx}{dt}\frac{du}{dx}dx.$$

We can also substitute $u = \frac{dx}{dt}$. Finally, we can rewrite the last two components of this integral as $\frac{du}{dx}dx = du$. Finally, we arrive at the solution:

$$W = \int mudu.$$

b. Show that

$$W = constant + \frac{1}{2}mu^2.$$

[Hint: the result from part 1 of this problem will be helpful]

SOLUTION
From the first part of this problem, we know that the derivative of $\frac{dE}{du} = mu$. In calculus, an *integral* represents the anti-derivative operation; in our

case, the anti-derivative of $\frac{dE}{du}$ is equal to $constant + \frac{1}{2}mu^2$. Thus

$$W = \int (mu)du = \int \frac{dE}{du}du = constant + \frac{1}{2}mu^2$$

Problem *SS-2* *(10 Points)*

A physical wave can be described mathematically by the function:

$$\Phi(x,t) = A\cos(2\pi x/\lambda - 2\pi f t),$$

where A is the amplitude of the wave (the maximum height above or below zero that the wave can reach), x and t are locations along the direction of propagation and time since the start of propagation (respectively), and λ and f are the wavelength (distance between two peaks or two troughs in the wave) and the frequency (rate at which two peaks pass the same point in space) of the wave. This function, $\Phi(x,t)$, (the *wave function*) thus tells you the amplitude of the wave at any point in space and at any time.

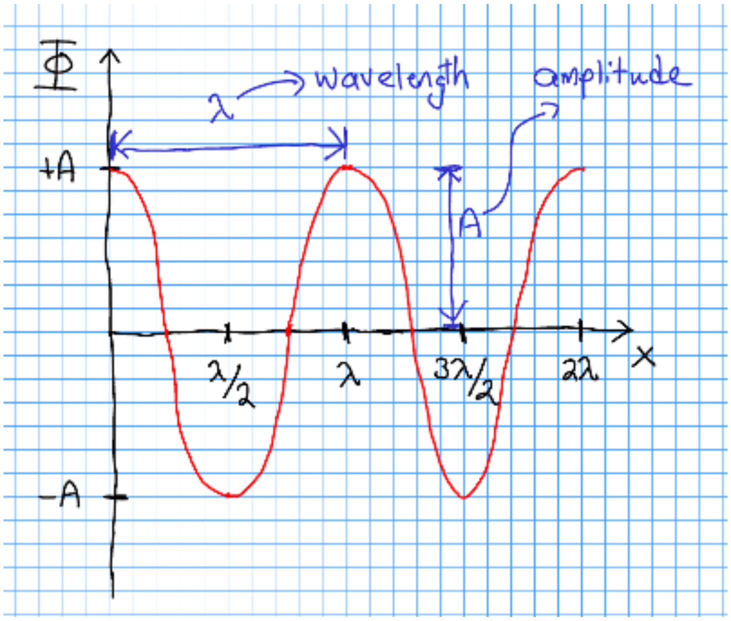
1. Draw the wave function from $x = 0$ to $x = 2\lambda$, represented as Φ vs. x (this means drawing two axes on your plot, where the vertical axis represents the value of Φ at a given point along the horizontal axis, and the horizontal axis represents a location in x). Label the amplitude, A, and the wavelength, λ, on your drawing of the function.
   SOLUTION
   Begin by thinking about the function and the values it takes at key points. In general, the cosine function takes its maximal or minimal values, ± 1.0, when its argument is equal to 0.0, π, and 2π (integer multiples of π). The function repeats itself once its argument reaches even multiples of π (e.g. 2π, 4π, etc.). In order to make a drawing of Φ vs. x, you must consider the spatial distribution of Φ at a specific time, t. That is because Φ is a function of t as well as x, but we are only interested in Φ vs. x. To keep things simple, let us consider the case $t = 0$ (considering other values of t will shift the points where the cosine is maximal away from integer multiples of π). Now we only need to know when the argument is equal to integer multiples of π. To figure that out, we must solve the equation:

   $$2\pi x/\lambda = n \times \pi \ (n = 0, 1, 2, ...).$$

   We can solve for x and find:

   $$x = \lambda \times n/2.$$

   We now know that the cosine reaches its maximal value when $n = 0, 2$ and its minimal value when $n = 1$, and these places occur when $x = 0$ (maximal), $x = \lambda/2$ (minimal), and $x = \lambda$ (maximal), and then it repeats. Plotting the function for $x = 0, 2\lambda$ means we should see two cycles of the function in the plot. So, finally, we can draw the function:



2. Waves are solutions to an equation known as the *wave equation*. This equation is written as:

$$\frac{1}{u^2}\frac{d^2\Phi}{dt^2} = \frac{d^2\Phi}{dx^2}.$$

Using that equation, and the wave function, solve for the velocity, u, at which the wave propagates.

SOLUTION
This problem requires you to take the second derivative of the wave function. It is useful to remember that

$$\frac{d}{dx}\sin(x) = -\cos(x)$$

and that

$$\frac{d}{dx}\cos(x) = \sin(x).$$

We can write the second derivative as the act of taking the first derivate twice in succession,

$$\frac{d}{dx}\left(\frac{d}{dx}f(x)\right).$$

With that in mind, let us evaluate the second derivative of the wave function with respect first to x and then to t:

$$\frac{d^2\Phi}{dx^2} = \frac{d}{dx}\left(\frac{d\Phi}{dx}\right) = \frac{d}{dx}\left(\frac{d}{dx}A\cos(2\pi x/\lambda - 2\pi ft)\right) = \frac{d}{dx}\left(\sin(2\pi x/\lambda - 2\pi ft)\frac{d}{dx}(2\pi x/\lambda - 2\pi ft)\right)$$

$$= \frac{d}{dx}\left(\sin(2\pi x/\lambda - 2\pi ft)(2\pi/\lambda)\right) = (2\pi/\lambda)\frac{d}{dx}\sin(2\pi x/\lambda - 2\pi ft) = (2\pi/\lambda)^2(-\cos(2\pi x/\lambda - 2\pi ft))$$

where in the last step the derivative of the sine function, then its argument, are taken all at once rather than showing the steps as in the derivative of the cosine function. We can then repeat this calculation for the second derivative with respect to t instead of x (steps will be skipped here, but they are similar to those shown above for the x derivative):

$$\frac{d^2\Phi}{dt^2} = (2\pi f)^2(-\cos(2\pi x/\lambda - 2\pi ft))$$

We now can write the wave equation as

$$\frac{1}{u^2}\left[(2\pi f)^2(-\cos(2\pi x/\lambda - 2\pi ft))\right] = (2\pi/\lambda)^2(-\cos(2\pi x/\lambda - 2\pi ft))$$

The cosines cancel from each side, as do the $(2\pi)^2$ and the minus signs. This then leaves

$$\frac{1}{u^2}f^2 = \frac{1}{\lambda^2}$$

and solving for u we find

$$u = \lambda f,$$

which is the statement that the speed of propagation of a wave is equal to the product of the wavelength and the frequency of the wave.

Harris Conceptual Questions

These refer to "Conceptual Questions" from a chapter or chapters in Harris.

- *CH2-2* (5 Points)

  SOLUTION
  Let us consider the "tractable, if somewhat unrealistic . . . " possibility that in order for Anna to actually cause the light bulbs to flash as the same time, her brain sends light signals to her hands. In reality, the nervous system transmits the command from the brain to muscles in the hand via electrical impulses, but the idea is the same: the command originates in the brain and is transmitted to both hands by signals propagating at the same speed (the speed of light, for this problem) over the same distance. So, how does this help address the criticism that "It makes no sense that Anna could turn on lights in her hands simultaneously in her frame but that they don't turn on simultaneously in another . . . "? Well, if the signals go from one place in her brain to each of her hands, one signal is sent along the eastward direction (toward the left hand) and the other along the westward direction (toward the right hand). In her frame, these signals are sent at the same time and travel at the same speed to

her hands, which then act on the signals at the same time. But to Bob, who sees Anna moving eastward at speed ν, he sees one light signal traveling at c toward the left hand, *moving away* at ν, while the other signal moves toward the right hand, which is *approaching* at ν. Thus the rate at which the distance closes between signals and hands is $c-\nu$ for the left hand and $c+\nu$ for the right hand. Since $c+\nu > c-\nu$, the signal going to the right hand arrives first, and thus that bulb is told to fire first. Light signals have to travel at c; however, simultaneous emission in one frame does not guarantee simultaneous arrival in another frame.

- *CH2-3* (5 Points)

  SOLUTION
  Each observer, in their frame, agrees on the magnitude but not the direction of relative motion. In other words, if the coordinate systems are defined so that $+x$ and $+x'$ point in the same direction, and the relative motion is along those parallel axes, frame S sees frame S' moving in the positive direction along x with speed ν, while frame S' sees frame S moving along the $-x'$ direction at speed $\nu' = -\nu$. So only one speed is necessary; and only the sign differs from frame to frame.

- *CH2-4* (5 Points) *[HINT: keep in mind the definition of proper time when thinking about this question*]

  SOLUTION
  In order to answer these questions, it is useful to recall the definition of *proper time*. Proper time is the time that is measured by an observer in a frame where all of the events occur at the same place. Is there such a frame in this problem? The answer is "yes". The events in question are comparisons of the clocks on the ground with the clock on the hang-glider. There are two of these events, one as the hang-glider passes over point X and the second as the hang-glider passes over point Y. Are the ground observers in the Proper Time Frame? No - these two events occur at different spatial locations for them. How about the hang-glider? Yes - both the person in that frame and the observers on the ground agree that the two events happen at the same place in the frame of the hang-glider. Now we can can answer the questions. Answer to (a): the clocks on the ground advance faster than yours, because in your frame events happen at the same place and so your time is *proper time*, the shortest time any frame will observe. Thus, your clocks will advance slower than those on the ground, where events happen in different locations. Answer to (b): Since your frame is the proper time frame, and clocks advance slower in that frame than those on the ground, the clock at Y must be ahead of yours. Answer to (c): it might seem natural for you, in the hang-glider, to argue that the ground is in motion and you are at rest - therefore, your first inclination might be that the ground clocks will be running slow. However, special relativity tells us that when events

happen in the same place in a frame - and for the hang-glider, they do - that time must be the shortest, and thus its clocks run slow. Observers in both frames will agree that for you, events happen in the same place; thus you are forced to conclude your time is the shortest.

Alternatively, you can argue directly from the Lorentz Transformation equations in each frame. In frame S, ground observers calculate the time measured by the hang-glider by applying

$$dt' = \gamma_\nu(-\nu/c^2 dx + dt).$$

For ground observers, the distance between their clocks at X and Y is given by the product of the hang-glider's speed (ν) and time it takes the hang-glider to pass between them in frame S, dt. Thus we can reduce the equation to

$$dt' = \gamma_\nu \cdot dt(-\nu^2/c^2 + 1).$$

Recognize that the quantity in parentheses is equal to $1/\gamma_\nu^2$, and you see that we get the same answer as in the Proper Time argument:

$$dt' = dt/\gamma_\nu \equiv dt_0,$$

Looking at this from the perspective of frame S', where all events happen at the same place ($dx' = 0$) and where the points X and Y on the ground are moving in the $-x$ direction at speed $-\nu$, we find

$$dt = \gamma_\nu(\nu/c^2 dx' + dt') = \gamma_\nu dt',$$

or that the hang-glider determines that clocks on the ground will measure more time than in frame S'. The frames make consistent observations of the relative states of their clocks.

Harris Exercises

These refer to "Exercises" from a chapter or chapters in Harris.

- *CH2-20* (10 Points)
  - **SOLUTION** It always helps to first think about which Lorentz Transformation equation you want to apply to the problem. Based on the way the problem is phrased, it's natural to decide that you are in frame S, at rest with respect to Carl who is in frame S' moving at $\nu = 0.5c$. We are interested in determining how much time passes for Carl, knowing the time that passes on your clock. Thus the equation we want to apply is the one in Carl's frame, relating his change in place and time to your change in time:

    $$dt = \gamma_\nu(\nu/c^2 dx' + dt').$$

    For Carl, he doesn't move at all so for him $dx' = 0$. His frame thus defines the frame where the shortest time, Proper Time, is defined. The time that passes on your clock is $dt = 1\,\text{min.}$. Thus the Lorentz Transformation equation reduces to

$$dt = \gamma_\nu dt'.$$

We just need to calculate the gamma factor: $\gamma_\nu = 1/\sqrt{1-(0.5c/c)^2} = 1.15$. We can now complete the calculation:

$$dt' = 1\,\text{min.}/1.15$$

yielding the solution to the problem,

$$dt' = 0.9\text{min}.$$

Remember, significant figures are enforced in the numerical answer. The two numbers given in this problem - 1 and 0.5 - each have only 1 sig. fig. Thus, their product (or division) must have only 1 sig. fig., so you can only quote to 1 decimal place.

- *CH2-21* (10 Points)
  - **SOLUTION**: Again, we need to begin by specifying frames S and S'. The way the problem is phrased, it's natural to assign to S the earth observer and to S' the spaceship, in relative motion with respect to S at speed $\nu = 0.6c$. Before we apply the appropriate Lorentz Transformation equation, let's compute the gamma factor: $\gamma_\nu = 1.25$. The length of the craft is measured by simultaneously ($dt = 0$) locating the ends of the ship in frame S, and in that frame the observer finds $dx = 35m$. The correct Lorentz Transformation equation to apply is thus

    $$dx' = \gamma_\nu(dx - \nu dt) = \gamma_\nu dx = 1.25 \times (35m).$$

    Thus, the people on the ship measure the length of the craft to be

    $$dx' = 40m.$$

    Again, while 35m has two sig. figs., the speed has only one. Thus, the final answer will have only one sig. fig., so you can't say $44m$ - you have to write $40m$. As a note, you can also argue this problem using proper length. Since the ship is at rest in S', the length must be the *longest* in that frame, and the length measured in S must be multiplied by γ_ν.

- *CH2-22* (15 Points)
  - **SOLUTION** Let's again start by assigning observers to frames. Let us place Bob in frame S and Anna in frame S', in relative motion to one another at speed $\nu = 0.8c$. According to the way the problem is phrased, Bob (taking into account the fact that it takes time for light from the explosion of Planet

Y to reach him) determines that Planet Y exploded two years after Anna passed Earth.

- Answer to (a): In two years, Anna would have traveled $(0.8c) \times 2\,\text{years} = 1.6\text{years} \cdot c = 1.6\text{ly}$ in that time. Thus the distance remaining from her to Planet Y, in frame S, would be 3.4ly. We can use Proper Length to solve the rest of this problem. The frame in which the *Earth-Planet* Y system is at rest is S, so in that frame we expect to measure the longest distance. Anna's frame, where the Earth and Planet Y are NOT at rest, must measure a shorter distance. The relationship is given by $dx' = dx/\gamma_\nu$. The gamma factor in this problem is $\gamma_\nu = 1.67$. This then gives us the answer

$$dx' = 3.4\text{ly}/1.67 = 2\text{ly}$$

.

- Answer to (b): Let's apply the Proper Time to solve this. All events happen at the same location in Anna's frame. Thus her time must be the shortest of any measured, and is related to times in other frames by $dt_0 = dt/\gamma_\nu$. The solution to the problem is thus

$$dt' = 2\text{yr}/1.67 = 1\text{yr}.$$

- *CH2-24* (15 Points)
  - **SOLUTION** Let's repeat the given facts. The pole-vaulter holds a 16ft. pole. A barn has doors that are 10ft. apart, one at each end. The runner achieves a constant speed before reaching the barn, and when he passes through the barn the pole appears to fit exactly in the barn all at once.
    - Solution to (a): To determine how fast the pole-vaulter is running, we can apply Proper Length. The frame in which the pole is at rest is the pole-vaulter's frame, so in all other frames the length of the pole will be shorter. Thus $L_0 = \gamma_\nu L$, where L_0 is the proper length. We can rewrite this equation to solve for ν:

$$L_0/L = \gamma_\nu$$

$$(L_0/L)^2 = \gamma_\nu^2 = \frac{1}{1 - \nu^2/c^2}.$$

Again, we rearrange the equation and solve for ν:

$$\left(1 - (L/L_0)^2\right) c^2 = \nu^2$$

$$\nu = c\sqrt{1 - (L/L_0)^2}.$$

We arrive at our solution:

$$\nu = 0.8c.$$

- Solution to (b): Since the pole is moving with respect to, for instance, a pair of observers standing at each end of the barn, to measure the length the people on the ground have to make *simultaneous* measurements of the locations of the ends of the pole. If the statement is made that the pole fits in the barn all at once, that statement can only be true in the rest frame of the barn and observers, since that is the frame in which the length is measured by simultaneously locating each end.

- Solution to (c): The pole vaulter is the one who sees the pole ends reach doors at different times. The pole-vaulter sees the barn moving relative to them, so the barn's length is contracted in that frame. That means that the barn is shorter than 10ft. in the pole-vaulter's frame, so the front end of the pole exits the barn before the back end of the pole enters the first door.

- Solution to (d): Let us first determine the length of the barn in the pole-vaulter's frame. Again, let's apply Proper Length. The barn is at rest in the earth's frame, so that is the frame where the length of the barn is greatest (and defined as the proper length, L_0). Thus the vaulter measures the length to be $L = L_0/\gamma_\nu = (10ft.)/(1.6) = 6.35ft.$. When the front of the pole exits the barn, this occurs when the back of the pole is sticking out the first door. The amount of pole left is $16ft - 6.35ft = 9.75ft$. Since the barn is moving past at $0.78c$ (keeping a few more decimal places for right now), the amount of time it takes until the back end of the pole enters the barn is

$$dt = (9.75ft.)/(0.8c) = (2.97m)/(0.8 \times 2.995 \times 10^8 m/s) = 1.2 \times 10^{-8} s.$$

$$dt = 1 \times 10^{-8} s$$

Problem *SS-3* (*15 Points*)

The 24 satellites that orbit the earth and define the Global Positioning System use precision atomic clocks that are accurate to 1×10^{-9}s, or 1 nanosecond (1ns). A person on the earth, using their mobile phone, can usually "see" 4-12 of the 24 satellites at any one time. The satellites are moving, relative to the surface of the earth, at 1.4×10^4 km/h.

1. Calculate the γ_ν for one of these satellites. It will be helpful to use the *Binomial Expansion* of the gamma factor,

$$\gamma_\nu = 1/\sqrt{1 - \frac{\nu^2}{c^2}} = 1 + \frac{1}{2}\frac{\nu^2}{c^2} + \frac{1}{8}\frac{\nu^4}{c^4} + ...$$

since the numbers involved in the calculation will be quite a lot smaller than 1. Feel free to write your answer either as a sum of terms in the binomial expansion or as $\gamma_\nu - 1$.

SOLUTION
A satellite is moving at a speed of $\nu = 1.4 \times 10^4$ km/h relative to the surface of the Earth. This speed is very small compared to that of light, so we cannot just plug this into a calculator and expect to get a meaningful answer. Instead, let's apply the Binomial Expansion, which tells us that $\gamma_\nu = 1 + \frac{1}{2}\nu^2/c^2 + ...$ The higher-order terms are negligible compared to the second term, and the second term tells us how much the gamma factor deviates from 1. We convert 1.4×10^4 km/h to m/s and obtain 0.39×10^4 m/s. Writing the final answer as the first two terms in the expansion:

$$\gamma_\nu = 1.0 + 8.4 \times 10^{-11} + ...$$

2. Civilian GPS measurements have to be accurate at the level of 5-10m. How accurate must our knowledge of the clocks on the satellites be in order to achieve this level of accuracy in our position measurement? *[HINT: light signals are used by your phone and the satellites to perform the triangulation]*

   SOLUTION
   We can think about this problem entirely in the Earth frame. All that matters for the operation of the system is that (a) signals sent from satellites to Earth travel at c and (b) the accuracy of the clocks on the satellites matches the desired spatial accuracy on Earth. There are two ways to approach this problem. Since all we need to know if the range of time accuracy needed to achieve a range of space accuracy (5-10 meters), and since distance and time of light signals are related to the speed of light, you need only apply $\Delta x = c\Delta t$. Using the upper and lower bound on the space range (5 and 10 meters), we can compute the upper and lower bound on the time range:

$$\Delta t = 5m/c = 2 \times 10^{-8} s \text{ to } \Delta t = 10m/c = 3 \times 10^{-8} s,$$

   or an accuracy of 20-30 nanoseconds. The clocks themselves are accurate to 1 ns, and the accuracy needed for civilian GPS is about 20-30 times bigger than the design of the clocks (good engineering!).

   The other way to attack this problem is using propagation and calculation of uncertainties on measured quantities (e.g. as learned in the PHY 1105 lab class, http://www.physics.smu.edu/~scalise/mechmanual/). The equation relating distance, time, and speed for light signals is $x = ct$. If the space measurement has an uncertainty of σ_x, then we can relate that directly to an uncertainty on the time measurement through the very precisely measured speed of light. This leads to $\sigma_x = c\sigma_t$, and we can again solve for the target range of time accuracy.

3. If we measure the passage of 1 day on earth using a clock that is identical to

the clocks on the GPS satellites, how much time have the clocks on the satellites measured? What about for the passage of 2 days on earth? For this problem, it will be useful to apply the Binomial Expansion again, this time to

$$\sqrt{1-\frac{\nu^2}{c^2}} \approx 1 - \frac{1}{2}\frac{\nu^2}{c^2}$$

SOLUTION
Consider the clock on a single satellite. All readings of the satellite's clock happen in the same location in this frame, so the satellite's frame defines the Proper Time frame. Again, the gamma factor is very, very close to 1.0 in this problem, so we need to employ the Binomial Expansion again. Writing the Proper Time (time in the satellite frame) as $dt_0 = dt/\gamma_\nu$, we can write the right-hand side as the product of time in the earth frame and the first two terms of the Binomial Expansion of $\sqrt{1-\nu^2/c^2}$:

$$dt_0 = dt\left(1.0 - \frac{1}{2}\nu^2/c^2 - ....\right)$$

We already solved for the numerical value of that second term, so we can write:

$$dt_0 = (1day) \times \left(1.0 - 8.4 \times 10^{-11} - ...\right)$$

So on the satellite, the clock is slower by 8.4×10^{-11} days, or 7.3 microseconds (7300 nanoseconds). In two days, or 172,800 seconds on earth, the effect is doubled to a time offset of 15 microseconds!

4. Is the difference between measured time on earth and on the satellites a problem for GPS position measurements? (in other words, do you need to take into account special relativistic effects?)

   SOLUTION
   Yes, it's a HUGE problem. In order to give you the accuracy needed to find, for instance, the correct house on a street, GPS MUST be accurate down to 20-30 nanoseconds. However, the special relativistic effect on the satellites' clocks makes them "drift" from identical clocks on earth by an additional 7.3 microseconds PER DAY! That's a drift which is about 200 times WORSE than the needed clock accuracy. In other words, each day the satellites are off by an additional 2km from the correct position on earth.
5. Based on what you have learned in this problem, if humans had never discovered special relativity before launching GPS satellites into space would we be able to actually use the GPS system?

   SOLUTION
   NO. If humans had started launching satellites in the 1800s, with the goal of using light signals from the satellites to triangulate positions on the earth, they would have found mysterious drifts in the reported locations of positions on earth. Special Relativity is thus an integral part of our modern lives, and

without knowledge of that effect of motion on space and time we would be unable to navigate with any precision at all using satellites and clocks.