Modern Physics (PHY 3305) Lecture Notes

HomeworkSolutions002

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Point Distributions

no tags

Points were distributed as follows for each problem:

Problem	Total	Point Distribution
CH2-14	20	Recognized that $E^2 = (mc^2)^2 + (pc)^2$ is the correct equation to apply (10 Points) and then correctly reasoned that photons represent energy, and lost energy represents a change in mass (10 Points)
SS-4	20	Part 1: Applied the fully relativistic Doppler Shift formula (3 Points) and obtained correct answers from the calculation (2 Points) Part 2: Applied the results from Part 1 to the frequency and the bits/second (3 Points) and obtained correct numbers for the scale of the effects (2 Points) Part 3: Applied a clear argument based on Part 2 to defend their answer (3 Points) and estimated the scale of loss of bits over the course of many transmissions (regardless of how many they assumed) (2 Points) Part 4: Applied an argument based on the Doppler Shift equation (3 Points) and chose to adjust the angle (2 Points).
CH2-65	20	Part a: Applied the transformation of velocities to the x and y components (5 Points), recognizing that both components of motion are affected (5 Points), and obtained correct answers from the application (5 Points) Part b: Obtained correct answers from the application (5 Points)
CH2-81	20	Recognized that the correct approach is to apply $E = mc^2$ (10 Points), reasoned that the total energy emitted from the sun (per unit time) was needed (5 Points), and correctly computed the result (5 Points)
SS-5	20	Part 1: Applied $E = mc^2$ to the problem (6 Points) and obtained correct answers (6 Points) Part 2: Reasoned correctly based on the comparison of

results from Part 1 to the given data (4 Points)
Part 3: Reasoned correctly given answers to the first two parts of the problem (4 Points)

Deductions outside of the above:

- 1 point is deducted for failing to box the numerical answer
- 1 point is deducted for incorrectly applying significant figures
- other points are deducted as outlined in the homework policy

HARRIS, *CH2-14* (20 Points)

SOLUTION

It is indeed true that in special relativity the photon is a massless particle traveling at the speed of light, whose energy is equal to $E = pc$. The sun is a massive body at rest, $p = 0$, whose total energy at any given time is given by its mass-energy, $E_{sun} = m_{sun}c^2$. If the sun is losing energy only through the emission of light, that would imply that

$$\Delta E = E_{sun} - \sum_{photons} p_{photon}c = \Delta m\, c^2,$$

and thus that over time the mass of the sun does change by an amount

$$\Delta m = (1/c^2)(E_{sun} - \sum_{photons} p_{photon}c).$$

PROBLEM *SS-4* (20 Points)

In 1997, the United States and European space agencies (NASA and ESA, respectively) launched a joint mission to the planet Saturn. The goal was to study the rings and moons of Saturn to better understand the evolution of the solar system. The satellite, named Cassini, contained on board a probe, named Huygens, designed to reach the surface of Titan, Saturn's largest moon. Titan was of particular interest because it contained the chemical building blocks of life - organic compounds - yet no one had ever seen beneath its thick methane clouds.

Huygens was launched from Cassini in December 2004 and 22 days later entered the atmosphere of Titan. Huygens was designed to send back video and audio of its journey. Data from Huygens was transmitted back to Cassini and relayed to Earth. The data was sent from Huygens using S-band radio waves, with a frequency range of 2.0-4.0 *GHz*, using a technique called "phase-shift keying" whereby information is sent from Huygens in packets at a rate of 8192 bits/second. The receiver on Cassini was tuned to receive S-band and the firmware on the satellite was written to expect information in packets that contained precisely 8192 bits/second; a large mis-match in frequency between the two, or the loss of bits from the packets, means catastrophic loss of data.

1. While Huygens itself is descending to Titan at a speed of about 200 km/h (relative to the surface of Titan), the original Cassini flight plan was designed so that it and the Huygens probe were moving away from one another at a MUCH larger speed - more like 10.0×10^3 km/h. What is the Doppler Shift on transmissions from Huygens back to Cassini at this relative speed? Which Doppler Shift is bigger: that due solely to relative motion, or the time dilation effect? *(HINT: Remember your Binomial Expansion.)*

   SOLUTION
   Huygens and Cassini are moving AWAY from one another, so $cos(\theta) = -1$ (because $\theta = 180\text{degrees}$) and the sign of the velocity in the Dopper Shift formula is positive instead of negative (e.g. since Huygens moves away from Cassini, the wavelength of signals it sends back to Cassini should be LONGER, and the frequency should thus be SMALLER. The denominator of the Doppler Shift must have a value greater than one for this to happen). Thus the equation relating emission frequency (f') on Huygens to reception frequency on Cassini (f) is

   $$f = f' \frac{\sqrt{1 - \nu^2/c^2}}{(1 + \nu/c)}.$$

   The relativistic contribution is given by $1/\gamma_\nu = \sqrt{1 - \nu^2/c^2}$, while the contribution due purely to motion away from one another is given by $1/\left(1 + \nu/c\right)$.

   We can now compute each of these contributions. It is useful to Binomial Expand the relativistic effect as

   $$\sqrt{1 - \nu^2/c^2} = 1 - \frac{1}{2}\nu^2/c^2 + ... = 1 - 4.30 \times 10^{-11} + ...$$

   We can also Binomial Expand the motion contribution as

   $$(1 + \nu/c)^{-1} = 1 - \nu/c + ... = 1 - 9.26 \times 10^{-6}.$$

   We see that the correction to 1.0 is much bigger from the motion effect than from the relativistic effect ($9.26 \times 10^{-6} > 4.30 \times 10^{-11}$). While both effects lead to a "red-shifting" of the radio waves from Huygens to Cassini, the motion effect dominates.

2. Is the frequency Doppler Shift enough to prevent the receiver from seeing the data? How does the Doppler Shift affect the number of bits/second that are received by the Cassini satellite (rounding to the nearest bit)?

   SOLUTION
   We need to evaluate how the Dopper Shift changes the range of frequencies covered by the S-band. The lower bound is 2.0 GHz and the upper bound is 4.0 GHz. Applying the results from the Doppler Shift in the last part of the problem, we can solve for how these boundaries on the frequency range are affected:

   $$f = (2.0GHz) \times (1 - 4.30 \times 10^{-11} + ...) \times (1 - 9.26 \times 10^{-6} + ...) \approx (2.0GHz) \times (1 - 9.26 \times 10^{-6}).$$

   So the shift in the frequency is about 1 part in 100,000 for both the lower and upper boundaries of the S-band, which is a very small effect and likely recoverable by the hardware and firmware.

   Let's now turn to the question of bit rate and loss of bits. If Huygens transmits 8192 bits/s, we can re-write this as $8192 \cdot \mathrm{bits} \cdot \mathrm{Hz}$, so we see this is just a multiplier of a frequency! We can plug this in for f' in the Doppler shift equation, and find the change in the bit rate:

   $$B \approx (8192\mathrm{bits/s}) \cdot ((1 - 9.26 \times 10^{-6}) = (8192 - 7.58 \times 10^{-2})\mathrm{bits/s}.$$

   If the loss is 0.076 bits/second, then one bit is likely to be lost every 13 packets (every 13 seconds). To set the scale, your typical digital image or sound file is several megabytes in size (a megabyte is about 8 million bits); that means one image would suffer the loss of at least 77 bits over the course of its transmission.

3. The design of the transmitter/receiver system actually failed to take into account the Doppler Shift. This oversight was discovered when Cassini passed by Earth on its way to Saturn. Would this mistake have the potential to doom the Huygens mission? Please briefly explain why based on your answers to the previous part of this question.

   SOLUTION
   The loss of even a single bit per unit of information can represent the catastrophic loss of data. Many bits are part of critical and irreplaceable units of information (e.g. "checkbits", or header information). Can losing 77 bits in a picture file be catastrophic? Think about what happens to a mobile phone video if the phone crashes or the battery dies, and the file does not close correctly - it's usually irrevocably corrupted.

4. The firmware could not be altered after launch. Pretend that you are an engineer working at NASA or ESA on the project, and you are under tremendous pressure to minimize the impact of the Doppler Shift *after* the satellite has already headed out to Saturn (after all, the project cost $3.26 billion and you can't let its success be threatened). Given that you cannot alter the receiver/transmitter design, hardware, or firmware (or any other

programming on that equipment) to address this issue, how would you adjust the *flight plan* of the satellite to minimize the impact of the Doppler Shift?

SOLUTION
Consider again the Doppler Shift equation:

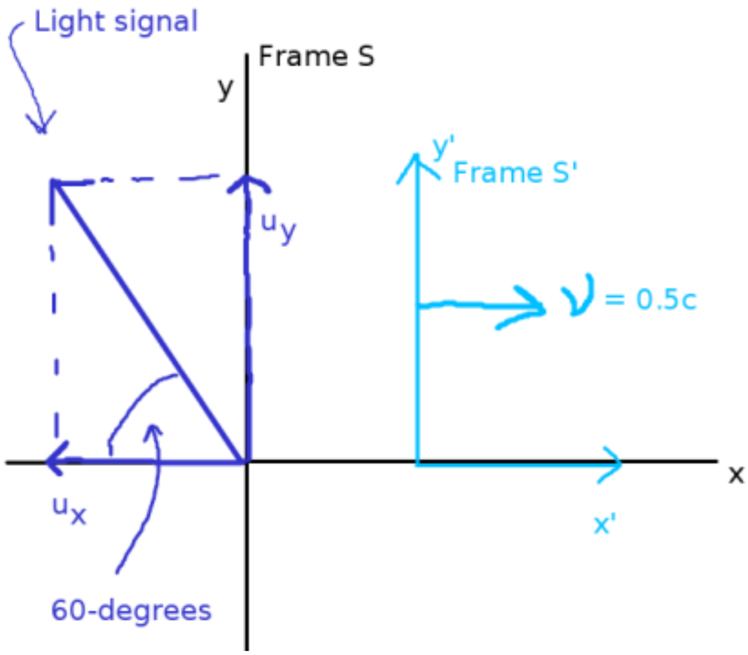
$$f = f'\frac{\sqrt{1-\nu^2/c^2}}{(1-\nu/c\cos(\theta))}.$$

If you're not allowed to adjust f', you're left with two options: change ν or change θ. Changing ν is possible, but given the high velocity of the Cassini satellite requires an immense amount of fuel to (a) bring nearly to rest and then (b) speed it back up again to complete its mission after dropping Huygens. Fuel is precious on space missions, so this is possible but likely to end the mission anyway. Instead, you could change θ by using a little fuel to adjust the DIRECTION of Cassini's flight, rather than the magnitude of its velocity. Changing θ to 90-degrees relative to the flight path of Huygens does the trick: $\cos(90\text{degrees}) = 0$, and so the Doppler Shift due to motion is gone, leaving only the Doppler Shift due to time dilation (which is four orders of magnitude smaller than that due only to relative motion). This was, in fact, what was done for Cassini in the real mission, and it saved most of the Huygens data. It took 2 years, while Cassini was in flight, to design and execute the change in flight plan!

HARRIS, *CH2-65* (20 Points)

SOLUTION

A picture helps here:



The above is what is described in part (a).

For part (a), we can decompose the components of the velocity of the light signal in Frame S. These yield:

$$u_x = -c \times \cos(60^\circ) = -0.50c$$

and

$$u_y = c \times \sin(60^\circ) = 0.87c.$$

Since Frame S' is moving relative to Frame S directly eastward at 0.5c, only the speed of the light signal projected along the x-axis will be affected by the motion. The transformation of velocity tells us

$$u'_x = \frac{u_x - \nu}{1 - u_x \nu / c^2}.$$

Plugging our numbers into the formula:

$$u'_x = \frac{-0.5c - 0.5c}{1 - (-0.5c \cdot 0.5c)/c^2} = -0.80c$$

(I'll accept 2 significant figures in the answers to this problem since Harris leads us to believe, largely by omission, that the angle of the beam is exactly 60 degrees and the speed is exactly half that of light).

The y-component in S' is given by:

$$u'_y = \frac{u_y}{\gamma_\nu \left(1 - u_x \nu / c^2\right)}$$

The gamma-factor here is $\gamma_\nu = 1.155$. We can then solve for the y-component of velocity:

$$u'_y = \frac{0.74c}{1.15\left(1 - (-0.66c)(0.5c)/c^2\right)} = 0.60c.$$

Note that the magnitude of the velocity of the signal in Frame S' is c - motion does not affect the total speed of the signal, even though the components appear to change due to motion.

For part (b), we have to reverse the sign of ν. We then find:

$$u'_x = 0.0c$$

$$u'_y = 1.0c$$

Again, the total speed of the signal is still c.

HARRIS, *CH2-81* (20 Points)

In order to determine the total change in mass of the sun, we need to determine how much energy the sun radiates. We are told that at the Earth's orbital radius $(1.5 \times 10^{11} m)$ sunlight arrives with an intensity of $1.5 kW/m^2$. That means that if we want to determine the total power radiated by the sun and arriving at that radius, we have to know the total surface area over which the light is spread. That area is the area of a sphere whose radius is $1.5 \times 10^{11} m$. From this we find

$$A = 4\pi R^2 = 2.8 \times 10^{23} m^2.$$

The total power emitted by the sun is then

$$P = (1.5kW/m^2) \times (2.8 \times 10^{23} m^2) = 4.2 \times 10^{23} kW$$

. Using $\Delta E = \Delta m\, c^2$, and dividing both sides by Δt, we obtain

$$(4.2 \times 10^{26} J/s)/c^2 = 4.7 \times 10^9 kg/s$$

.

As a note, given that the current mass of the sun is about $2.0 \times 10^{30} kg$. At the above rate, it would take 1.3×10^{13} years for it to "evaporate" away.

PROBLEM *SS-5* (20 Points)

Einstein's recognition that mass and energy are two aspects of the same thing helped to lead to the prediction of a new form of matter called "anti-matter." As its name implies, when anti-matter and matter meet they completely annihilate each other. All of their mass is converted into pure radiation (e.g. light).

1. Imagine that there is exactly one gram of anti-matter, and it contacts and completely annihilates with exactly one gram of matter. Assume that each gram is brought into contact very, very slowly, so that you can ignore momentum. How much energy is released? Compare that to the energy released by a hydrogen bomb, the most powerful weapon ever devised, whose explosion is equivalent to 50 million tons of TNT (1 ton of TNT is equivalent to 4.184×10^9 Joules of energy).

   SOLUTION
   We apply $E = mc^2$ to determine the energy generated by the complete annihilation of the 2 grams of matter and anti-matter. We find

   $$E = (0.002kg) \cdot c^2 = 1.8 \times 10^{14} J.$$

   This is equivalent to the detonation of about 1/1000 of a hydrogen bomb, or 43,000 tons of TNT.
2. In the movie and the book "Angels and Demons," a tiny amount of anti-matter is stolen from CERN, a global center for subatomic particle physics research. While CERN is real, and anti-matter is real, the technology featured in the movie which stores and isolates anti-matter is not real. However, accepting the premise, imagine that, as in the movie, a quarter of a gram of anti-matter is suddenly brought into contact above Vatican City and Rome. Should we have expected the story to continue - that is, would anybody have survived the event and gone on to find the killer?

   SOLUTION
   It's incredibly likely that about 10,000 tons of TNT going off in the skies over Vatican City and Rome would have done tremendous heat and concussive damage to the city and its inhabitants. For comparison, this is equivalent to the yield of "Little Boy," the atomic weapon that devastated Hiroshima, Japan, at the end of World War II.
3. Given your answers to the first two questions, is anti-matter abundant on Earth? Please explain your answer.

   SOLUTION
   If anti-matter existed in concentrations even at the level of a fraction of a gram anywhere in or on the earth, the detonation would be noticable (think about how "easy" it is to detect the underground detonation of atomic weapons given today's sensitive nuclear disarmanent measures). If such quantities were

produced by cosmic rays in the atomsphere, we'd be treated to quite a light show day and night! The lack of observational evidence for such emissions of energy suggests that anti-matter is present on Earth in extremely small quantities. It is regularly produced by cosmic rays and nuclear decays, for instance, but so rarely that the impact is usually a single anti-particle on its matter counterpart, yielding very little energy.