Modern Physics (PHY 3305) Lecture Notes

HomeworkSolutions003

SteveSekula, 16 February 2010 (created 15 February 2010)

Point Distributions

no tags

Points were distributed as follows for each problem:

Problem	Total	Point Distribution
CH3-2	10	Commented on both the observations regarding intensity and wavelength (4 Points) and employed an argument about how energy is transmitted by waves and particles to draw conclusions (6 Points). Appealing to things derived from experiments like this (e.g. that wavelength/frequency and energy are related for light) will not earn points, since they represent "circular reasoning" (e.g. such conclusions were DERIVED from this experiment, rather than use as input to explain this experiment, so employing them represents using conclusions to argue the point)
CH3-25	20	Recognized that the correct equation to apply was $KE = hf - \phi$ (5 points), correctly related kinetic energy to the stopping potential and wavelength to frequency (5 points), made a graph of the data (5 points), and reasoned that a line was the correct description and then used that to solve for Planck's constant (regardless of the answer obtained) (5 points)
CH3-36	20	Applied the Compton scattering formula to all parts of the problem (5 Points), correctly calculated the answers to parts (a) and (b) (10 points), and arrived at a reasonable conclusion in (c) based on work in parts (a) and (b) (5 Points)

SS-6	20	Part 1: Recognized how to relate the total intensity and the area of the sail to the number of photons striking the sail per second (5 Points) Part 2: correctly determined the change in momentum per photon (5 Points) and computed the total force on the sail per second (5 Points) Part 3: Applied the force equation again to determine the time for acceleration (5 Points).
CH4-4	10	Determined what the problem meant by "all things being equal" (7 Points) and applied that definition to answer the question (3 Points)
SS-7	20	Part 1: applied the force picture (7 Points) and calculated the velocity of the electron (3 Points) Part 2: computed the momentum (3 Points) and the wavelength (2 Points) Part 3: Compared the wavelength and relevant dimensions (4 Points) and drew a conclusion from the comparison (1 Point)

Deductions outside of the above:

- 1 point is deducted for failing to box the numerical answer
- 1 point is deducted for incorrectly applying significant figures
- other points are deducted as outlined in the homework policy

- HARRIS, *CH3-2* (10 Points)

  SOLUTION
  To deduce what is going on in this experiment, it is useful to reconsider the examples given in class of freeing a ball from a pond or from the branches of a tree. In fact, let's concentrate on the tree example. If freeing a ball from a tree is something best done by a wave phenomenon, then you could imagine standing at the base of the tree and shaking it. In general, the energy delivered by waves is proportional to intensity, while for particles it is related instead to the individual particle properties (momentum, energy). Intense waves, regardless of the properties of each singular wave, should eventually

build up enough energy to knock the ball loose. In contrast, if the tree is too rigid to shake and only hitting the ball loose with another ball will work, you can stand alone and throw balls at the tree hoping you eventually hit it - as long as your throw delivers sufficient energy to the stuck ball, it can be freed. Adding more pitchers to the problem (increasing the particle intensity) multiplies the probability that any one ball will strike the struck ball and knock it immediately loose, but everything hinges on the kinetics of the individual thrown balls.

Let's apply this thinking to the problem at hand. The energy delivered to the electrons in the metal appears to have nothing to do with intensity and everything to do with wavelength. For waves, we expect intensity and energy to be directly related. This experiment opposes that notion, and suggests that the light behaves instead like a collection of particles, whose energy is related to its wavelength and not its intensity.

- HARRIS, *CH3-25* (20 Points) *if you write a computer program to solve this, or devise a computational method to solve this, you must turn in the code/files used to obtain the answer*

  SOLUTION I have solved this problem two ways, to illustrate different approaches to the solution. The first is a hand-drawing of the line, with solutions for the slope of the line, and the second is the application of a spreadsheet and line-fitting in a computer program (*OpenOffice*).

  Both solutions require some fundamental thinking about the data that are given. The data are wavelengths of light and the stopping potential. Let us consider Einstein's explanation of the photoelectric effect:

  $$KE = hf - \phi.$$

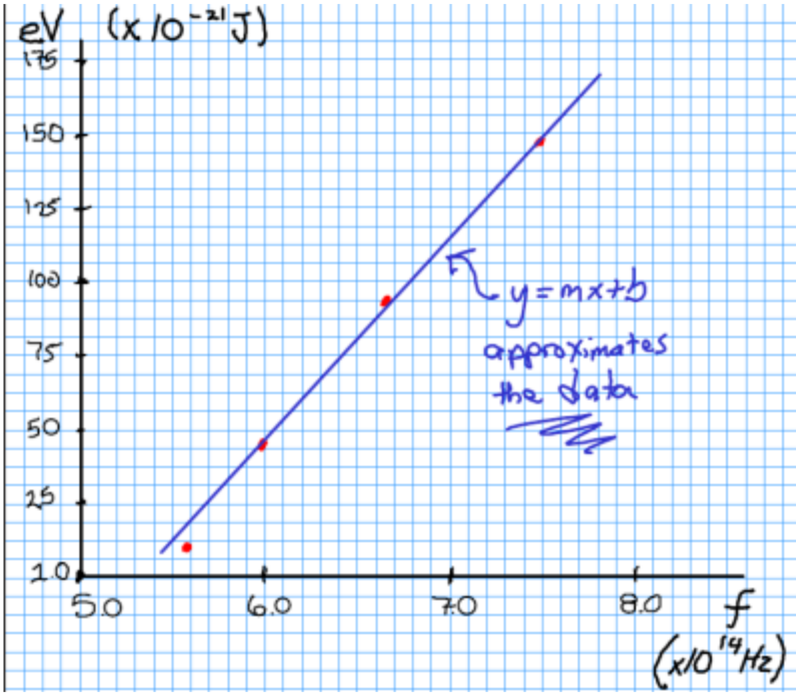
  This explanation says that the maximum kinetic energy any ejected electron can have is equal to the difference between Planck's constant times the frequency of light and the work function of the metal. The work function is not given in the problem, but as we'll see this is not an issue. We have the frequencies of the light - we need only transform the wavelengths into frequencies via $f = c/\lambda$. We obtain the following frequencies (all in units of 10^{14} Hz): 5.45, 5.99, 6.66, and 7.49.

In addition, we are given the stopping potential. How does this relate to something in Einstein's equation? We know that the stopping potential is that voltage required to bring the flow of electrons in the experiment to zero; since the light is freeing electrons from the metal, bringing the current means stopping their motion by creating an opposing force (the electric force) to their motion - thus the potential energy of the electric field, eV (where e is the charge of the electron) must be equal to the highest kinetic energy available to the electron, $hf - \phi$. Therefore, we have $eV = KE$, and we can now plot KE vs. f to see the relationship in the data. The values of eV for the four data points are (in units of 10^{-21} J): 9.60, 45.76, 90.08, and 145.28.

The data on the y-axis range from about 1×10^{-21} J to about 150×10^{-21} J, and on the x-axis from about 5×10^{14} Hz to 8×10^{14} Hz. Thus we can draw out plot with these boundaries on these axes and sketch the points on the plot.

The final piece of this, after you plot the data (see below), is to observe that a line is a good approximation of the trend in the data. The equation for a line is $y = mx + b$, where m is the slope of the line and b is the y-intercept of the line. This equation looks exactly like the photoelectric effect equation, where $h = m$ and $-\phi = b$. You can use a line-fitting routine in a spreadsheet, or you can average slopes of each segment of the line (or other methods of estimating the slope), to arrive at your answer.

For a spreadsheet-based solution, see http://www.physics.smu.edu/sekula/phy3305/photoelectric_data.xls



- HARRIS, *CH3-36* (20 Points)

  SOLUTION
  Answer to part (a): The maximum possible change in wavelength for photon-electron scattering is given by the Compton Scattering formula,

  $$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

  when $\cos\theta = -1$ (in other words, complete back-scatter of the photon, $\theta = \pi$). In this case, the maximum change in wavelength is given by the constant $2h/m_e c = 4.9 \times 10^{-12} m$, using $m_e = 9.1 \times 10^{-31}\text{kg}$, $c = 2.995 \times 10^8\,\text{m/s}$, and $h = 6.63 \times 10^{-34}\text{J} \cdot \text{s/m}$.

  Answer to part (b): the maximum now is still for backscatter, but we must replace the mass of the electron with the mass of the proton: $m_p = 1.7 \times 10^{-27}\text{kg}$. This yields $\Delta\lambda = 2.7 \times 10^{-15}\text{m}$.

  Answer to (c): Which more clearly demonstrates the particle nature of electromagnetic radiation? In order to see particle behavior, the wavelength of the light has to be short enough that the wave is seen to lose significant energy, and thus be seen to act more like a scattered particle than a wave. Clearly, the electron has a bigger absolute effect on the wave properties, and thus is more likely to cause the particle behavior of light to become apparent.

PROBLEM *SS-6* (20 Points)

A "solar sail" is a large, lightweight, highly reflective device which has been proposed as a means for cheap transport in space (one is featured in Star Wars: Attack of the Clones - it is Count Dooku's escape ship). The sail is unfurled and reflects starlight. The theory is that since light carries momentum, its reflection off the sail exerts force on the sail and pushes the ship.

1. The light emitted by our sun is "white light" - that is, it contains all visible light wavelengths. Make the simplifying assumption that the wavelength is 550nm - the middle of the visible spectrum. If the power of sunlight reaching the Earth is $1.5\mathrm{kW/m^2}$, how many photons per second hit a solar sail that has an area of $0.25\mathrm{km}^2$?

   SOLUTION
   The intensity of the light reaching an Earth-orbit is given, and represents the power (energy per unit time) received per unit area. You dealt with this quantity in your previous problem about the change in mass of the sun. Knowing that the energy per photon is given by $E_\gamma = hf = hc/\lambda$, we can use that to determine the photons per second per unit area. Including the area of the sail then lets us compute the photons per second arriving on the sail. Plugging in numbers, we obtain

   $$I = 1.5\mathrm{kW/m^2} \times 0.25\mathrm{km^2} = N_\gamma E_\gamma = N_\gamma hc/\lambda$$

   where N_γ is the number of photons per second. We can then solve for this and find

   $$N_\gamma = 1.0 \times 10^{27}\mathrm{photons/s}.$$

2. If the solar sail reflects all incident light completely, what force would sunlight exert on the sail?

   SOLUTION
   The force exerted on the sail will be equal to the change in momentum

of the sail per unit time, $F = \Delta p/\Delta t$. Let us conveniently choose the unit of time to be 1 second, since in one second we know how many photons strike the sail. We need to solve for the change in momentum of the solar sail, and the only way we can do this is to apply momentum conservation. Let us define $\vec{q}_i$ and $\vec{q}_f$ as the initial and final momentum of the solar sail. Let us also define as the direction of positive motion as that along which the light is originally traveling. The initial momentum of the sail if zero, and the final momentum will have a positive sign if it is non-zero. Let us denote the initial photon momentum (of a single photon) as positive and equal to $\vec{p}_i = +hc/\lambda_i$, and the momentum after bouncing perfectly off the solar sail as $\vec{p}_f = -hc/\lambda_f = -p_f$. We have two unknowns: $\vec{q}_f$ and λ_f. We can write down the momentum conservation equation along the axis of motion as

$$0 + p_i = q_f - p_f.$$

We need a second equation to help solve for the unknowns in the first equation. We can use energy conservation. The initial energy of the solar sail is Mc^2, where M is the mass of the solar sail, and the final energy is $Mc^2 + \frac{1}{2}M\nu^2$, which we can rewrite as $Mc^2 + q_f^2/2M$, using $q_f = M\nu$. The initial energy of the photon is $p_i c$ and the final energy is $p_f c$. We can then write energy conservation as

$$Mc^2 + p_i c = Mc^2 + p_f^2/2M + p_f c$$

which then simplifies (canceling the Mc^2 terms) to

$$p_i c = q_f^2/2M + p_f c.$$

From this, we can solve for the unknown quantity p_f

$$p_f = p_i - q_f^2/(2Mc)$$

and substitute that into the momentum conservation equation,

$$p_i = -p_i + q_f^2/(2Mc) + q_f.$$

We can rewrite this as a quadratic equation in q_f and solve for q_f:

$$q_f^2 + (2Mc)q_f - (4Mc)p_i = 0.$$

Applying the quadratic equation solution,

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where in our case $a = 1$, $b = 2Mc$, and $c = -4Mc$, we find:

$$q_f = \frac{-2Mc \pm \sqrt{4M^2c^2 + 16Mcp_i}}{2}.$$

We can rewrite this as

$$q_f = Mc\left(-1 \pm \sqrt{1 + 4p_i/(Mc)}\right)$$

and using the binomial expansion of the square-root, $\sqrt{1+x} = 1 + \frac{1}{2}x + ...$, we obtain

$$q_f = Mc\left(-1 \pm (1 + 2p_i/(Mc) + ...)\right)$$

The only physical solution to this problem is the positive one, and we obtain:

$$q_f = 2p_i.$$

We are now equipped to determine the force. The force is the change in momentum over the change in time. In 1 second, 1.0×10^{27} photons strike the sail and bounce off, and the momentum change from each photon is $2p_i$. So the total change in momentum in one second gives

$$F = (1.04 \times 10^{27}\mathrm{photons/s}) \times (2h/\lambda_i) = 2.5\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2 = 2.5\mathrm{N}.$$

3. Assume that the mass of such a solar sailing ship is 9000.0 kg. If you could sustain the power on the sail at the level of $1.5\mathrm{kW/m}^2$, how long would it take to accelerate the ship to 0.5c?

   SOLUTION
   If we could sustain this intensity of light, then we would be putting a constant force of $2.5\mathrm{N}$ on the sail. Using $F = \Delta p/\Delta t$, we can determine how long it would take to accelerate a 9000.0 kg solar sail to 0.5c. We can solve the force equation for Δt, using $\Delta p = \gamma M \Delta \nu = (1.155) \times (9000.0\mathrm{kg}) \times (0.5c) = 1.56 \times 10^{12}\mathrm{kg} \cdot \mathrm{m/s}$ We obtain:

$$\Delta t = \Delta p/F = (1.56 \times 10^{12}\mathrm{kg} \cdot \mathrm{m/s})/(2.5\mathrm{N}) = 6.2 \times 10^{11}\mathrm{s}$$

or about 17,000 years.

- HARRIS, *CH4-4* (10 Points)

  SOLUTION
  All other things being equal, an electron is more likely to exhibit its wave nature. We have to define what "all other things being equal" means. What can we make equal, besides the two things which are not (their respective masses)? Velocity is really the only candidate. If we set the velocity of the particles to be the same, ν, then we can compare the wavelengths of the particles:

  $$\lambda_p = h/(m_p \nu)$$

  and

  $$\lambda_e = h/(m_e \nu).$$

  This demonstrates that when all else is equal, the electron is more wavelike (has a larger wavelength) because $m_e < m_p$. In order to make one as wavelike as the other, you would need to slow down the proton relative to the electron. Making the speed unequal, $\nu_p < \nu_e$, will compensate for the masses being different.

PROBLEM *SS-7* (20 Points)

SOLUTION

This problem is a variation of Harris, *CH4-25*.

The question of whether or not we can treat a single atom using classical physics, or whether we should apply quantum physics, is important to making progress on understanding and manipulating atomic structure. We will try to address this question here.

1. Consider a Hydrogen atom. This is composed of a proton at the center

(the nucleus), orbited by an electron. Let us think about the atom classically for a moment. If we are to treat the atom classically, we must think of the electron in orbit around the proton, attracted to it by the Coulomb force. How fast must the electron be moving in order to maintain the measured distance between the electron and the proton, 0.1nm?

SOLUTION
There are two forces at play in the classical hydrogen atom picture: the Coulomb force and the centripetal force. The forces are balanced if the electron is to remain in orbit. In that case,

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_pQ_e}{r^2} = \frac{m_e\nu^2}{r}.$$

Here, r is the radial orbit distance of the electron from the proton, $Q_e = Q_p$ are the charges of the electron and proton, m_e is the mass of the electron, ν is its orbital speed, and $\varepsilon_0 = 8.85\times10^{-12}F/m$ is the permittivity of free space. We can rewrite this and solve for the electron's speed:

$$\nu^2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_e^2}{m_e r} \rightarrow \nu = 1.6\times10^6\mathrm{m/s}$$

or about $5\times10^{-3}c$, which is quite non-relativistic.

2. What is the wavelength of this electron?

   SOLUTION
   The wavelength can be determined from the momentum,

   $$\lambda = h/p = h/(m_e\nu) = 4.6\times10^{-10}\mathrm{m} = 0.5\mathrm{nm}$$

3. In order to treat the atom as a classical object, its wavelength must be smaller than the dimensions of its orbit; if the wavelength is comparable, or larger, then we cannot think of the electron as a particle orbiting another particle. Compare the wavelength of the electron to the radius of its orbit. Can we treat the Hydrogen atom classically?

   SOLUTION
   The wavelength of the electron is slightly larger than the radius of its orbit, or more accurately comparable to the circumference of its orbit, which is about 0.6nm. We cannot, therefore, expect the atom to behave as a classical bound state of an electron particle orbiting a proton

particle; instead, we must carefully consider the matter wave of the electron and its behavior around the proton.