

Modern Physics (PHY 3305) Lecture Notes

HomeworkSolutions008

SteveSekula, 13 April 2010 (created 11 April 2010)

Point Distributions

no tags

Points were distributed as follows for each problem:

Problem	Total	Point Distribution
<i>CH10-10</i>	10	Noted that resistance is related to impurities in a lattice and that zinc is like an impurity in the copper (5 Points). Realized that if temperature doesn't have a big effect on resistivity, then impurities, not thermal motion, must be the dominant effect (5 Points)
<i>CH10-14</i>	10	Recognized that thermal motion is the dominant factor at high temperature (5 Points) but that in semi-conductors thermal energy can promote more electrons to the conduction band (5 Points)
<i>CH10-17</i>	10	Noted that control of semi-conductor properties is the dominant effect (6 Points), through band structure (2 Points) and conductor contributions (2 Points)
<i>CH10-21</i>	10	Identified a Cooper pair as a bound state of two electrons (8 Points) and recognized that bosons can all be in the same state and act as one (2 Points)
<i>CH10-48</i>	15	Part (a): recognized that the whole crystal width is the potential width (5 Points) Part (b): recognized that the top of a band has a wavelength and lattice spacing relationship (2 Points) which can be used to solve for the energy (2 Points) to find that lattice spacing is the right scale (1 Points) Part (c): commented on answer to part (b) (5 Points)

<i>CH10-50</i>	20	Used the wavelength/band position relationship (10 Points) to find the energy of the top of the two bands (8 Points) and solved for the approximate band gap (2 Points)
<i>CH10-52</i>	15	Computed the energies of visible light (10 Points) and realized that the large band gap in carbon makes it impossible for carbon to absorb visible-light photons (5 Points) while silicon readily absorbs such photons (5 Points)
<i>SS-11</i>	10	Part 1: Recognized that if yellow, orange, and red light are not absorbed then green, blue, indigo, and violet light must promote electrons across the gap (5 Points) and used that to estimate the band gap (3 Points). Part 2: using answers from part 1, made a relative argument between copper and gold (3 Points) which resulted in copper having a smaller band gap (2 Points)

Deductions outside of the above:

- 1 point is deducted for failing to box the numerical answer
- 1 point is deducted for incorrectly applying significant figures
- other points are deducted as outlined in the homework policy

HARRIS *CH10-10*

SOLUTION

The zinc in brass acts as a contaminant in the otherwise dominantly copper lattice. The fact that brass's resistivity is significantly higher than either copper or zinc suggests that electron interactions with zinc atoms are disruptive to their motion. The other fact - that decreasing temperature doesn't have a large effect on brass' resistivity - reinforces this observation that impurities are the major player in brass' resistance.

HARRIS *CH10-14*

SOLUTION

What factors decrease the conductivity of a conductor as temperature increases? Thermal motion of the ions is the most significant contribution, increasing as temperature increases. These factors are also present in a semi-conductor. However, in a semi-conductor as temperature increases MORE charge carriers are promoted across the gap to the conduction band, increasing conductivity and countering the increase in thermal ionic motion.

HARRIS [CH10-17](#)**SOLUTION**

Why are doped semi-conductors so pervasive in modern technology? The intrinsic semi-conductor has properties fixed by the inherent structure of its bands and gaps. Doped semi-conductors allow for control of those properties, but altering gaps and providing additional conductors (holes or electrons). It is the tailoring of physical properties to the needs of modern devices that makes doped semi-conductors so incredibly useful.

HARRIS [CH10-21](#)**SOLUTION**

A Cooper pair is a weakly bound pair of electrons, whose binding occurs because of an attraction through lattice interactions. Since they are boson states they can easily occupy the same state and act as one, allowing free motion without resistance across the body of the lattice.

HARRIS [CH10-48](#)**SOLUTION**

Part (a): The box length L would be the entire width of the crystal, since in this model we treat the sum of all ionic potentials as one large potential with n representing the number of anti-nodes in the wave function of a single electron. The n quantum number plays here the same role it played

in the infinite square well - it labels the energy of the state, which is DIRECTLY related to the number of anti-nodes in the wave function.

Part (b): The top of band n is marked by a wave function whose wavelength is EXACTLY equal to some integer, m , times π , divided by the lattice spacing, a . In this case, the number of antinodes is instead the total number of atoms, N , times the integer labeling the band - nN (consider Fig. 10.25 in Harris, where this relationship is directly presented). The width of the crystal (W) is AGAIN the appropriate length scale, and the energy would be given by:

$$E = \frac{(nN)^2 \pi^2 \hbar^2}{2mW^2} = \frac{(n)^2 \pi^2 \hbar^2}{2m(W/N)^2} = \frac{(n)^2 \pi^2 \hbar^2}{2ma^2}.$$

So we see that the lattice spacing, $a = W/N$, plays the role of the potential well width in this view of the crystal where BANDS are labeled by n , rather than labeling the original single-well energy levels by n .

Part (c): *Explain why the energies of the band do or do not depend on the size of the crystal as a whole.* We see from part (b) that the energy of a band, n , is NOT related to the size of the crystal but, instead, is related to the lattice spacing (the distance between any two ions in the lattice). One ionic potential is enough to describe the more complex properties of an entire lattice; in solid-state physics, you can make TREMENDOUS progress simply by thinking about a single unit of a lattice. In our case, this is a single ion (1-D potential). In 3-D, this becomes a unit of the crystal. This is an incredible property, and makes much easier the prediction of properties of a material.

HARRIS [CH10-50](#)

SOLUTION

Again, we begin by referring to Fig. 10.23 and 10.25 in Harris. We see that the top of the $n = 2$ band is marked by cells of width a corresponding to a WHOLE WAVELENGTH of the electron wave function. Thus, $a = \lambda$. The top of the $n = 1$ band is marked by a cell size that is HALF a wavelength, or $a = \lambda/2$. If we assume that the energy of an electron is approximately the free-particle energy,

$$E = \frac{\hbar^2 k^2}{2m},$$

then we can determine the energy of an electron at the top of a given band:

$$E = \frac{\hbar^2}{2m} \left(\frac{n\pi}{a} \right)^2.$$

The energy difference between the top of these two bands is

$$\Delta E = \frac{\hbar^2}{2m} (2^2 - 1^2) \left(\frac{\pi}{a} \right)^2 = \frac{3\hbar^2\pi^2}{2ma^2} = 50 \text{ eV}.$$

This is an over-estimate, since the bottom of the $n = 2$ band does not correspond to the top of the $n = 1$ band. However, it's a reasonable estimate of this band width.

HARRIS [CH10-52](#)

SOLUTION

Let us think about this problem by considering white light striking the crystals in question (carbon and silicon). White light consists roughly equally of all wavelengths between infrared and ultraviolet light (700nm-400nm). Let us compute the equivalent range of energy corresponding to this range of wavelengths. Doing so (using $E = hc/\lambda$) we find the corresponding energy range to be 1.8eV-3.1eV.

In carbon (diamond), the band gap is 5eV. Consider what happens when visible-light photons strike conduction band electrons. The photons are incapable of moving valence electrons into the conduction band, and so carbon will not absorb visible light photons. These photons pass through the carbon, unaffected by the electrons in the material.

In silicon, the band gap is only 1 eV. Any visible light photon easily promotes valence electrons across the gap and into the conduction band. Silicon absorbs all visible light shining on it, and thus appears black (opaque) to our eyes.

Problem [SS-11](#) (worth 10 of the 25 Points awarded jointly to [CH10-52](#) and this problem)

As a "final part" of Harris [CH10-52](#), explain:

1. Gold has one conduction electron and appears yellow-orange to our eyes, which means gold is very good at absorbing and re-emitting red, orange, and yellow light, but is not good at absorbing and re-emitting green, blue, or violet light. Based on this, estimate the band gap in gold. *HINT: a cut-off in which wavelengths are most readily re-emitted suggests that photon energies above yellow cause electrons to enter a state from which they can return only by emitting a well-defined photon.*



SOLUTION

If gold is easily re-emitting yellow, orange, and red light, but not green, blue, indigo, or violet light, then it suggests that the band gap lies at an energy just ABOVE the energy of yellow light. Yellow light has a wavelength of about 575nm, corresponding to an energy of 2.2eV. Thus we can infer that the band-gap in gold is somewhere just above 2.2 eV. In fact, precise measurements of the band gap in copper reveal it to be around 2.3 eV.

2. Based on your understanding of the answer to Part 1, explain whether the band gap in copper (which also has one conduction electron but appears more orange in color than gold) is larger or smaller than in gold.



SOLUTION

Since copper appears more red-orange, that means that the band gap must be SMALLER in copper. Yellow light is capable of promoting valence electrons across the gap to the conduction band, and is thus readily absorbed by copper.